# The Story of the Solution of the Cubic and the Birth of Imaginary Numbers

The last decades of the 1400s and into the 1500s saw increased intellectual activity (Renaissance). The Guttenberg printing press came out in 1450, universities in Bologna, Paris and Oxford sprung up, Michelangelo painted the 16th chapel (1510), and da Vinci (1452-1519) was dabbling in a number of fields. Columbus had sailed the ocean blue and civilization (and the knowledge with it) began to expand.

Our story begins in 1494 when Italian Luca Pacioli produced *Summa de Arithmetica* ("A Summary of Arithmetic"), a piece of work focusing on solving linear and quadratic equations, doing so 100 years before the symbolic algebra you now use in school. In this, he claims that the cubic equation ($ax^3 + bx^2 + cx + d = 0$) was impossible to solve, which set forth a challenge.



*Figure 1 - Luca Pacioli*

In 1485, Scipione del Ferro finds a solution for $x^3 + cx - d = 0$ (no quadratic term), but keeps it a secret. He did so, because at the time, academic appointments (professorships) depended in part on political action and public mathematical challenges (scholarly battles/debates). He kept it a secret until his deathbed where he passed it along to his student Antonio Fior in 1526. Fior would use it in a 1535 challenge against Niccolo Fontana. Fontana was best known for a slash to the face he had received from a soldier who broke into his home when Fontana was 13. Fontana's speech suffered mightily from this mark and he became better known as Tartaglia (the Stammerer).



*Figure 2 - Tartaglia*

During the challenge, each sent 30 questions to the other. All of the problems Fior sent were of the form $x^3 + cx - d = 0$, so he put all of his eggs into one basket, betting that Tartaglia would fail. As the deadline for end of the battle approached, Tartaglia shuffled frantically until the night of February 13, 1535, when he found the solution and solved all of Fior's questions. Fior did miserable on his 30 questions and Tartaglia took his position.

Next enters Gerolamo Cardano (1501-1576). Cardano was born an illegitimate child and due to this illegitimacy, his mother took many medications to abort him. Most likely due to this, he had a number of health issues growing up (insomnia of up to 8 nights, extreme amounts of urine, heart palpitations, extreme fear of heights and dogs). He eventually became a physician, but was not allowed to practice in his hometown of Milan due to his peculiar personality. He was an ardent believer in astrology and superstitions, even predicting the future based off thunderstorms, and a huge gambler. He moved back to Milan in 1532 and began giving lectures at the university on medicine, religion and math that became popular. He was eventually offered a position at the College of Physicians in Milan in 1539 and soon became the most well-known physician in Europe, even treating the Pope!



Figure 3 - Cardano

The story returns because after Tartaglia became famous for his victory, Cardano continually pestered him to share his solution. Tartaglia (though not at first) eventually shared his solution, but made Cardano write an oath not to share.

With Tartaglia's work, Cardano was able to solve the general cubic equation ($ax^3 + bx^2 + cx + d = 0$) that Pacioli had originally claimed impossible. He published his findings in his book *Ars Magna* ( "The Great Art") 6 years later, crediting Tartaglia for some of it, but Tartaglia was furious. Three years later in 1548, one of Cardano's students, Ludovico Ferrari, defeated Tartaglia in a public debate by using Tartaglia's work against him and Tartaglia faded away from the public eye.

Cardano's method would also provide instances of taking square roots of negative numbers. He dismissed this as being "as subtle as it is useless" and disregarded it even though it would return solutions with real answers. It would be 100 years until Rafael Bombelli used imaginary numbers in his 1572 Algebra to show how they could be used to provide real solutions. He did not answer what they were, just how to operate with them.

The final words of Cardano's famous *Ars Magna* were "Written in five years, may it last as many thousands."