Taxicab Geometry 101

Thanks to
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Geometry starts with

- Points, Lines and Planes – undefined terms
- Then define a segment AB,
  \[ \{x : D(A,x)+D(x,B) = D(A,B) \} \]
- From this definition what does a segment look like.
Euclidean formula

d_{E}(A,B) = \sqrt{(a_1-b_1)^2 + (a_2-b_2)^2}

Euclidean segment
What is the Taxicab segment between the two points?

\[ dT(A,B) = |(a1-b1)| + |(a2-b2)| \]
Why do the taxicab segments look like these objects?
Taxicab Geometry

- There is no moving diagonally or as the crow flies.
- The movement runs North/South (vertically) or East/West (horizontally).
- Movement is similar to driving on streets and avenues that are perpendicularly oriented.
Which is closer to the post office? The Museum or City Hall?

City Hall because \( d_{T(P,C)} = 3 \) and \( d_{T(P,M)} = 4 \)
What does a Euclidean circle look like?

- Euclidean Circle
  \{dE(P,A) = 3\}
What does a Taxicab circle look like?

- Taxicab Circle
  \[dT(P, A) = 3\]
In Taxicab Geometry find \( d_T(P,A) = 3 \) and \( d_T(P,B) = 7 \) with \( A = (-4,2) \) and \( B = (2,-2) \).
In Taxicab Geometry find \( \{d_T(P,A)=5\} \) intersect \( \{d_T(P,B)=5\} \) with \( A=(-4,2) \) and \( B=(2,-2) \)
What does a Euclidean midpoint look like? (points, segment, midpoint)
\{ x : DE(A,B)=DE(A,x)+DE(x,B) \text{ and } DE(A,x)=DE(x,B) \}
Midpoints

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Midpoints

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Perpendicular bisectors

- What is the definition of a perpendicular bisector?

  The set of all points that are equidistant from two specific points, say A and B.
What are the Euclidean Perpendicular Bisectors?

\{ x : DE(A,x)=DE(x,B) \}
What are the TaxiCab Perpendicular Bisectors?

\{ x : DT(A,x)=DT(x,B) \}
What are the TaxiCab Perpendicular Bisectors?

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What are the TaxiCab Perpendicular Bisectors?

\{ x : DT(A,x) = DT(x,B) \}
A water fountain company wants to put water fountains in a 12 by 12 block park so that wherever people are they are within two blocks of a water fountain. Money is tight and the company wants to know where the water fountains should be placed so they can be the most cost effective.
Ali and Bryce have to walk to $A = (-3, -1)$ and $B = (3, 3)$ respectively. Where should they live so that the sum of the distances they have to walk is a minimum?
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Where should they live if they want the distances to be the same?
Bryce decides to be a gentleman and will make sure he walks farther than Ali. Where should they live?
Bryce found out that he has to be at work by 5:30 every morning and Ali doesn’ t have to be to her work until 8:00. Bryce wants to live closer to work so he doesn’ t have to get up as early to walk to work. Where should they live?
The Cement Company wants to locate its plant at a point P for which \( dT(P,A) + dT(P,B) + dT(P,C) \) is a minimum, where \( A = (-6,0) \) is a sand quarry, \( B = (5,0) \) is a boat dock, and \( C = (-2,5) \) is a railroad freight yard. Find P.
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There are three elementary schools in this area. What school should the students attend so they have the least distance to travel according to where they live?
The perpendicular bisector of A and C.
Here are the boundaries between C and B.
Here are the boundaries between A and B.
Here are all of them together.
Indiana attempted to assign a constant value to PI.

What is the definition of PI?

What is the value of PI in Euclidean geometry?

What is the value of Pi in TaxiCab geometry?

Text book: Taxicab Geometry E.F. Krause – Amazon 6.95
Textbook – Amazon $6.95

Geometers sketchpad constructions for
- Segment
- Circle
- Perpendicular bisector (?)

Tools to use to solve problems
Additional Explorations

- Taxicab parabola
- Taxicab ellipse
- Taxicab hyperbola
Summary

This is a new type Geometry for the students

The math solving part is only counting which makes it easier for the students who struggle in math

It will allow you to ask thoughtful and useful questions of every student

I plan on introducing this to my students when I teach    Alexis Wall, Amber Severson, …