Honors 1 Name: ___________________________________

Quadratics Unit 2 and Chapter 9 Note Packet

8.7 Day 1 – Factor ax² + bx + c

Review Examples:

1. $x^2 + 4x + 3$		Check:
2. $2x^2 - 2x - 24$		Check:

Thoughts: Why is factoring #1 easier than factoring #2? Can we always factor out the leading coefficient?

Steps to Factoring Any Quadratic

1. Factor out the leading coefficient.

2. Factor the remaining part ($x^2 + bx + c$) and do not simplify fractions until the very end.

1. $2x^2 + 11x + 12$		Check:
2. $3x^2 - 14x - 5$		Check:
3. $6x^2 + 5x - 4$		Check:

Use mental math to factor: You can think of factoring backwards and use mental math to factor as well.

$$4x^2 + 7x + 3$$

8.7 Day 2 – Factor and Solve $ax^2 + bx + c = 0$

Review Example: One can always use the fraction technique to solve these. However, a little number sense could help us reason through what the two factors must be in some simpler cases. Use the leading coefficient and constant term to try and figure out the factors, ***mentally***.

need to multiply to make 2

$3x^2 + 7x + 2 =$

( )( )

need to multiply to make $3x^2$

Examples: Solve the following by factoring first.

1. $2x^2 + x - 3 = 0$	2. $-x^2 + x + 20 = 0$

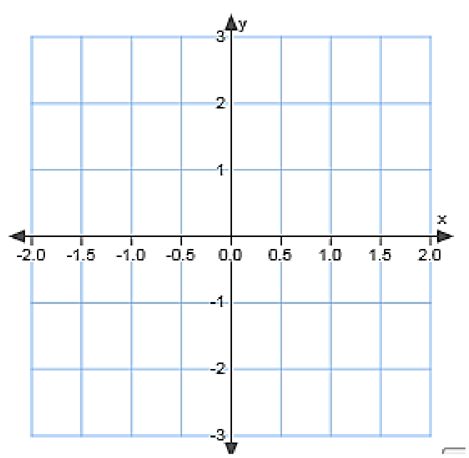
Factor using mental math to solve.

1. $5x^2 + 23x + 24 = 0$	2. $4x^2 - 13x + 10 = 0$

9.2b – Graph Quadratic Equations of the Form $y = ax^2 + bx + c$ by Factoring to Find Zeros

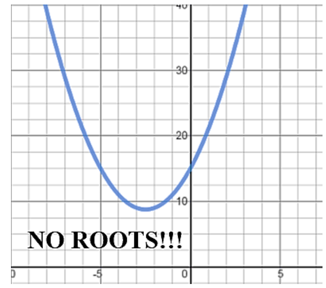
Example: Graph $y = 2x^2 + 3x + 1$

Step 1: Factor the equation	Step 2: Find the zeros
Step 3: Find the vertex	Step 4: Find the y-int and its reflection



9.1 – Graph Quadratic Equations That Don't Have Roots by Using –b/2a for the Vertex

Lead-In: When you tried to graph $y = x^2 + 5x + 15$ by factoring you couldn't. The reason why is because this quadratic doesn't have any roots, so it can't be factored! See graph to the right.



So, how are we going to graph quadratic equations that don't have roots? Well, we are going to use an old skill to find the vertex of **any** quadratic.

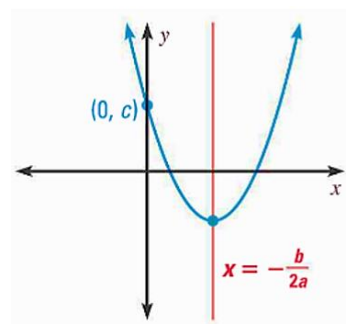
Example: Find the vertex of the function by first finding the x-value halfway between the x-intercepts.

1. $y = -2x^2 + 5x$	2. $y = ax^2 + bx$

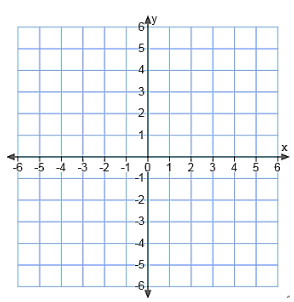
Key Finding: The x-coordinate of the vertex will always be located at ______________________________

Properties of a Quadratic Function y = ax^2 + bx + c:

1.) if a>0 then it opens up
2.) if a<0 then it opens down
3.) the axis of symmetry is at x = -b/2a
4.) Use the x-value of the axis of symmetry (the x-coordinate of the vertex) to find the y-value of the vertex.
5.) The y-intercept is (0,c)



Example

1. Find the axis of symmetry, the vertex, and y-intercept of the graph of $y = x^2 + 2x + 5$ 1. Axis of symmetry is 2. The x-coordinate of the vertex is 3. The y-coordinate of the vertex is 4. The y-intercept is 	**Vocabulary Terms:** *Maximum* *Minimum* **Example**: Tell whether the function $f(x) = -3x^2 - 12x + 10$ will have a minimum or maximum value. Then find that value.

9.3b – Graph Quadratics in Vertex Form

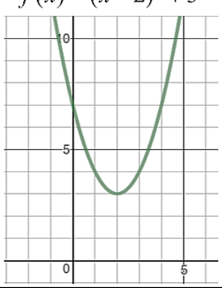
Lead – In: After the last quiz you conducted an investigation of another way to write a quadratic equation:

$$f(x) = a(x-h)^2 + k$$

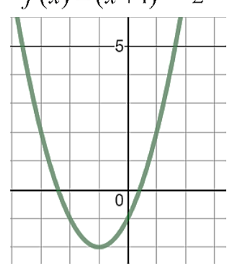
Parameter	Transformation to Parent Function $f(x) = x^2$
a	
h	
k	

Example: State the vertex for each graph. Do you see how it relates to its equation?

$f(x) = (x-2)^2 + 3$



$f(x) = (x+1)^2 - 2$



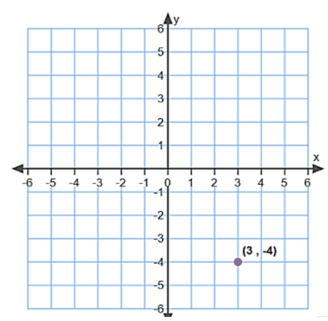
How to use "*a*" to Graph Outside of the Vertex

$g(x) = 2(x-3)^2 - 4$

x	
$g(x)$	

1) We know the vertex is located at (3,-4). What are two other values we could easily plug in for x to get two other points?

2) What is a shortcut way to use "a" to get two other points quickly, without having to plug anything in?



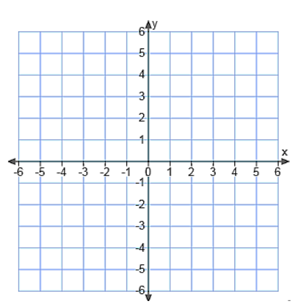
Steps to Graph in Vertex Form:

1)

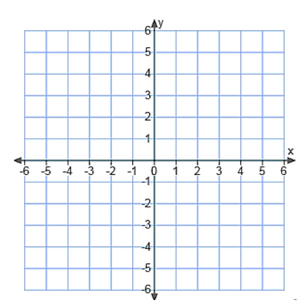
2)

3)

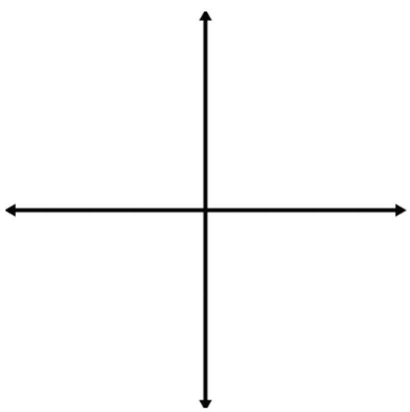
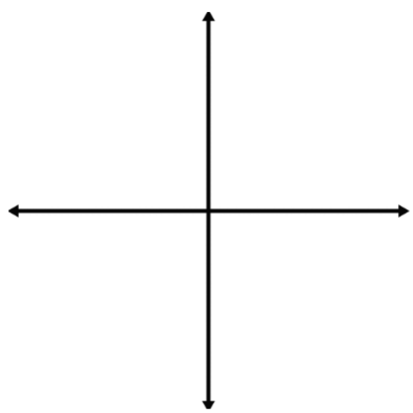
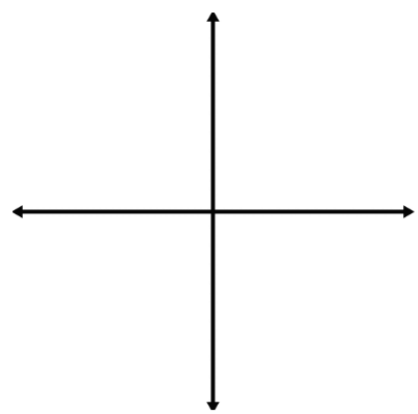
$f(x) = -3(x-5)^2 + 4$



$f(x) = 0.25(x+2)^2 - 1$



Three Forms for Writing and Graphing a Quadratic Function:

Standard Form $f(x) = ax^2 + bx + c$	**Factored From** $f(x) = a(x - m)(x - n)$	**Vertex Form** $f(x) = a(x - h)^2 + k$
Example	**Example**	**Example**
Graph: 	Graph: 	Graph: 

8.9 – Solve Quadratic Equations by Using Square Roots

Lead – In: if I asked you to find the x-intercepts of $y = 3x^2 - 27$ before, you would have to do it by factoring, like so:

Examples: Now, we are going to solve quadratic equations using a new tool: square roots.

Steps: 1) **2)** **3)**

$3x^2 - 27 = 0$	$p^2 + 12 = 12$	$2a^2 + 3 = -15$	$4z^2 = 9$	$3x^2 - 11 = 7$

Examples: Solve multi-step equations and use vertex form to find the roots (x-intercepts of a function).

$2(x-5)^2 = 18$	Take the following function written in ___________ form and find the roots of the quadratic function. $f(x) = -4(x+3)^2 + 16$

Applications of Square Roots: Estimating heights of locations by dropping items and measuring fall time.

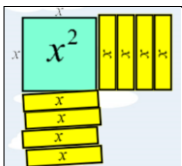
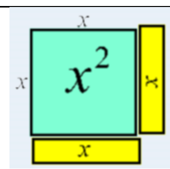
Recall, the height equation for items that move in the air is: $h = -16t^2 + vt + s$, where t is time (sec), v is initial upward speed (ft/sec) and s is initial height (feet). How long would it take an item to hit the water if dropped off the Golden Gate Bridge?

9.4 – Complete the Square

Lead-In: Multiply the following and see if you can discover the pattern.

$(x+3)^2$	$(x-4)^2$	$(x+7)^2$	$(x-5)^2$

Example: Fill in the statements to make the statement a perfect square.

 $x^2 + 8x +$ ____	 $x^2 + 2x +$ ____	$x^2 + 6x +$ ____	$x^2 - 10x +$ ____

Example: Now, use our new tool of completing the square to make a squared expression to solve the equation.

$x^2 + 6x = 7$		$x^2 - 8x = 5$	
Algebraically	Pictorally	Algebraically	Pictorally

Example: You can use completing the square it **anytime, to solve any quadratic equation.**

$x^2 + 5x = 10$

9.4 Day 2 – Solve by Completing the Square and Convert From Standard Form to Vertex Form

Example: Solve by completing the square.

$2x^2 + 5x - 8 = 0$

Steps to Convert From Standard Form to Vertex Form

1.	2.	3.	4.

Example: Convert to vertex form. $y = 2x^2 - 12x + 23$	**Example**: Convert to vertex form. $y = 3x^2 + 24x + 41$

9.5 – Solve Quadratic Equations by Using the Quadratic Formula

Lead-In: Our tool of completing the square will always work to solve ANY quadratic equation. On the board I have shown all the steps that are involved for solving even very involved ones. As you can tell, there are A LOT of steps.

So, what the ancient Babylonians thought of was instead of doing these steps **over and over** again, let's do them **once**, in general (this means using only variables), and make an equation!

QUADRATIC FORMULA: formula that can be used to solve any quadratic equation. However, it should only be used when all other tools fail, as the other tools (square roots, factoring, completing the square) are faster.

For any equation of the form: $ax^2 + bx + c = 0$, the solutions to the equation are

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Example: solve the equation $2x^2 - 4x - 3 = 0$ a = _______ b = _______ c = ______

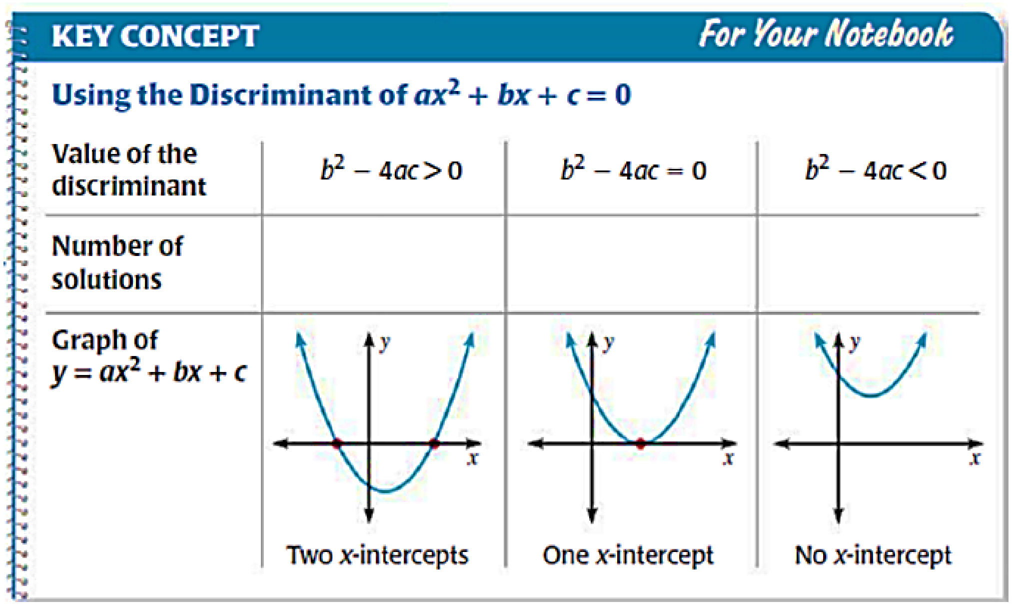
Example: solve the equation $3x^2 + 5x = 8$ a = _______ b = _______ c = ______

9.5 Day 2 – Program the Quadratic Formula and Interpret the Discriminant

Lead-In: Use the quadratic formula to find the x-intercepts of the following graphs.

$y = 2x^2 + 6x + 5$ a = _____ b = _____ c = _____ $x = \frac{-6 \pm \sqrt{6^2 - 4(2)(5)}}{2(2)}$ $x = \frac{-6 \pm \sqrt{-4}}{4}$	$y = x^2 - x - 7$ a = _____ b = _____ c = _____ $x = \frac{1 \pm \sqrt{1^2 - 4(1)(-7)}}{2(1)}$ $x = \frac{1 \pm \sqrt{29}}{2}$	$y = 4x^2 - 12x + 9$ a = _____ b = _____ c = _____ $x = \frac{12 \pm \sqrt{12^2 - 4(4)(9)}}{2(4)}$ $x = \frac{12 \pm \sqrt{0}}{8}$

Discriminant:



KEY CONCEPT *For Your Notebook*

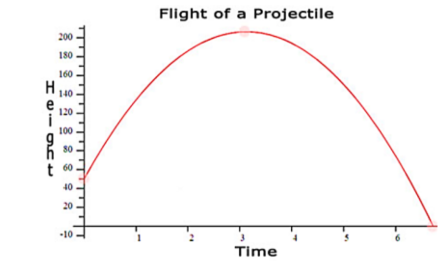
Using the Discriminant of $ax^2 + bx + c = 0$

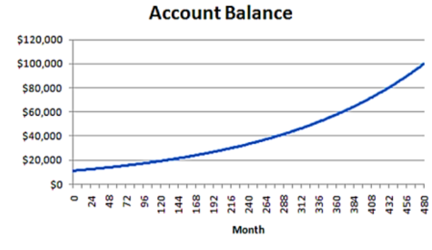
Value of the discriminant	$b^2 - 4ac > 0$	$b^2 - 4ac = 0$	$b^2 - 4ac < 0$
Number of solutions			
Graph of $y = ax^2 + bx + c$	Two x-intercepts	One x-intercept	No x-intercept

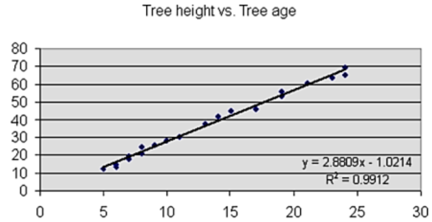
Example: Tell whether the equation $3x^2 - 7 = 2x$ has two, one or no solutions.

9.6 – Comparing Linear, Quadratic and Exponential Functions/Models

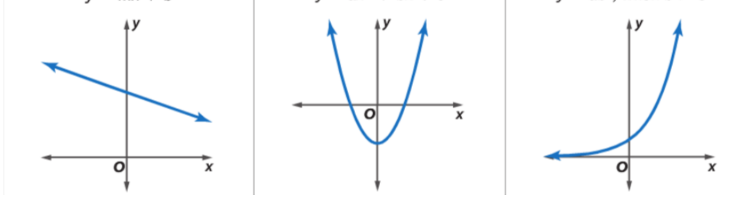
Which function would best model the following three situations?







________ Function	________ Function	________ Function
$y = mx + b$	$y = ax^2 + bx + c$	$y = ab^x$, when $b > 0$



Example: Determine what type of equation could model the following points and write an equation.

x	−2	−1	0	1	2
y	−8	−3	2	7	12

x	−1	0	1	2	3
y	8	4	2	1	0.5

x	−4	−3	−2	−1	0
y	32	18	8	2	0

Summary:

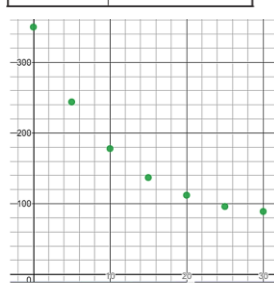
Linear if:

Exponential if:

Quadratic if:

Example: A baking company is testing its cakes to see how long they take to cool so they can tell consumers when to apply icing. They take a cake out of the oven and measure its temperature every 5 minutes.

Time (min)	Temperature (°F)
0	350
5	244
10	178
15	137
20	112
25	96
30	89



Use a calculator to find a regression fit (linear, quadratic or exponential) for the data

1) Type your data into L1 and L2 by going to STAT -> EDIT

2) Plot your data somehow

3) Based on plot, the most appropriate model is ________________ because:

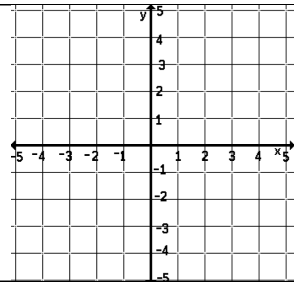
4) Use the appropriate regression and estimate the temperature of the cake after 12 minutes.

9.7 – Absolute Value and Piecewise Functions

Make a table and graph the following.

$f(x) = |x|$

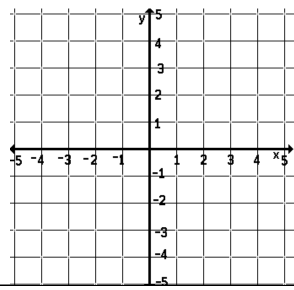
x	f(x)



Write the piecewise equation for $f(x) = |x|$.

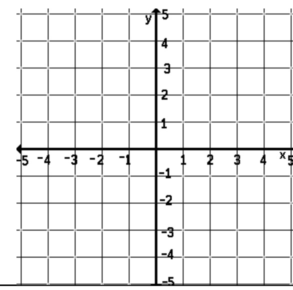
$f(x) = |x - 1| + 2$

x	f(x)



$f(x) = 2|x + 3|$

x	f(x)



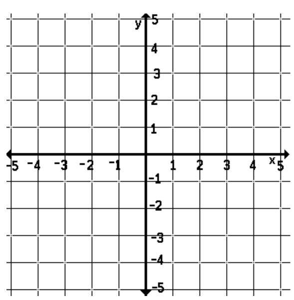
Piecewise-Defined Function:

Example: Graph the following piecewise functions or write their equation.

$$f(x) = \begin{cases} -2x - 3, & x \leq 1 \\ 4, & x > 1 \end{cases}$$

Domain:

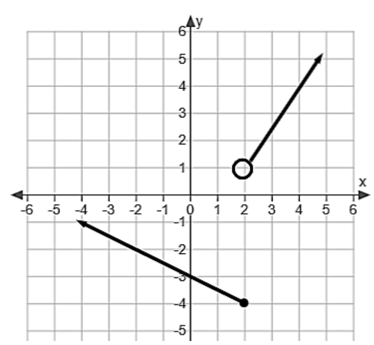
Range:



$$f(x) = \begin{cases} \quad\quad\quad , \\ \quad\quad\quad , \end{cases}$$

Domain:

Range:



Application Example: A cell phone company charges \$50 for your first 5 GB of data. After that, it charges a rate of \$10 per GB over 5 GB. Write the piecewise function for this situation.

$$f(x) = \begin{cases} \quad\quad\quad , \\ \quad\quad\quad , \end{cases}$$

Now, use it to calculate the cost if you use 6.5 GB in a month.