Name: ___________________________________ Per:_____

Investigation into the Birth of Imaginary Numbers

Answer the following questions based on the reading

1. What did Luca Pacioli claim was impossible to solve?
2. How could you take someone's position as a professor back in this day?
3. Where did Tartaglia get his name?
4. Who did Tartaglia share his solution with?
5. Why did Tartaglia fade from the public's eye?

Investigation

In Algebra 1 and Algebra 2 you learned the solution to the quadratic: $ax^2 + bx + c = 0$. We call it the quadratic formula and it looks like:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

This solution was known to the Babylonians as early as 2000 B.C.

6. Use the quadratic formula to solve $2x^2 + 7x + 5 = 0$.

Note: a = 2, b = 7, c = 5.

$$x = \frac{-7 \pm \sqrt{7^2 - 4(2)(5)}}{2(2)}$$

$$x = \frac{-7 \pm \sqrt{49 - 40}}{4}$$

$$x = \frac{-7 \pm \sqrt{9}}{4}$$

$$x = \frac{-7 \pm 3}{4}$$

$x = \frac{-7+3}{4}$ and $x = \frac{-7-3}{4}$

$\boldsymbol{x = -1}$ and $\boldsymbol{x = -2.5}$

In the reading you learned that Cardano discovered the cubic version of the quadratic formula (that is, the formula to solve equations of powers of 3: $ax^3 + bx^2 + cx + d = 0$). Cardano's formula is shown below:

$$x = \sqrt[3]{\left(\frac{bc}{6a^2} - \frac{d}{2a} - \frac{b^3}{27a^3}\right) + \sqrt{\left(\frac{bc}{6a^2} - \frac{d}{2a} - \frac{b^3}{27a^3}\right)^2 + \left(\frac{c}{3a} - \frac{b^2}{9a^2}\right)^3}}$$
$$+ \sqrt[3]{\left(\frac{bc}{6a^2} - \frac{d}{2a} - \frac{b^3}{27a^3}\right) - \sqrt{\left(\frac{bc}{6a^2} - \frac{d}{2a} - \frac{b^3}{27a^3}\right)^2 + \left(\frac{c}{3a} - \frac{b^2}{9a^2}\right)^3}} - \frac{b}{3a}$$

7. Use Cardano's formula to attempt to solve the equation $x^3 - x^2 - 2x = 0$. At some point you will be trying to take the square root of a negative number. Stop when you get to that point.

Note: a = 1, b = -1, c = -2, and d = 0. For simplicity, note the following first:

$$\left(\frac{bc}{6a^2} - \frac{d}{2a} - \frac{b^3}{27a^3}\right) = \left(\frac{(-1)(-2)}{6(1)^2} - \frac{0}{2(1)} - \frac{(-1)^3}{27(1)^3}\right) = \left(\frac{2}{6} - 0 + \frac{1}{27}\right) = \frac{10}{27}$$

$$\left(\frac{c}{3a} - \frac{b^2}{9a^2}\right) = \left(\frac{-2}{3(1)} - \frac{(-1)^2}{9(1)^2}\right) = \left(-\frac{2}{3} - \frac{1}{9}\right) = -\frac{7}{9}$$

Then, Cardano's formula would simplify to the following in this case:

$$x = \sqrt[3]{\left(\frac{10}{27}\right) + \sqrt{\left(\frac{10}{27}\right)^2 + \left(-\frac{7}{9}\right)^3}} + \sqrt[3]{\left(\frac{10}{27}\right) - \sqrt{\left(\frac{10}{27}\right)^2 + \left(-\frac{7}{9}\right)^3}} - \left(-\frac{1}{(3)(1)}\right)$$

$$x = \sqrt[3]{\left(\frac{10}{27}\right) + \sqrt{-\frac{\mathbf{1}}{\mathbf{3}}}} + \sqrt[3]{\left(\frac{10}{27}\right) - \sqrt{-\frac{\mathbf{1}}{\mathbf{3}}}} + \frac{1}{3}$$

This is as far as we can go with his formula at this point.

Intro to Imaginary Numbers

The interesting part about Cardano's formula is it led to solutions even when trying to take the square root of negative numbers. For example, if you use his formula to solve $x^3 - x^2 - 2x = 0$, one arrives at a situation where they are trying to take the square root of a negative number. Typically, if we have the square root of a negative number, we would say there is "NO SOLUTION." This made sense, until someone noticed that x = 2 is a solution.

8. Verify that indeed x = 2 is a solution to the equation $x^3 - x^2 - 2x = 0$.

$$(2)^3 - 2^2 - 2(2) = 0$$
$$8 - 4 - 4 = 0$$
$$0 = 0$$

We will see how imaginary numbers were invented and used in upcoming lessons