Name: ____ Per: ________

Introduction to Polar Coordinates

In one sense it might seem odd that the first way we are taught to represent the position of objects in mathematics is using Cartesian coordinates when this method of location is not the most natural or the most convenient.

When you ask a child where they left their ball they will say "just over there'' and point. They are describing (albeit very roughly) a distance: "just'' and a direction: "over there'' (supported by a point or a nod of the head). When you ask someone where their town is, they often say things like "about 30 miles north of London''. Again, a distance and direction. It is not very often that someone gives the latitude and longitude of their location!

Below is a map of Montana. Correctly identify which town I am describing based on the rough directions given with Helena as the home base (origin).

1. Town 75 miles and 60° north of east of Helena: Great Falls
2. Town 80 miles and 55° south of east of Helena: Bozeman
3. Town 45 miles and 33° west of south of Helena: Butte
4. Town 175 miles and 50° north of east of Helena: Havre
5. Town 380 miles and 7° north of east of Helena: Glendive

Now, you try by giving a description of where the following are in reference to Helena.

1. Salmon, Idaho: 130 miles and 45° south of west of Helena **or** 130 miles and 45° west of south of Helena
2. Cody, Wyoming: 200 miles and 45° south of east of Helena **or** 200 miles and 45° east of south of Helena
3. Missoula, Montana: 95 miles and 15° north of west of Helena **or** 95 miles and 75° west of north of Helena

You may have noticed already, I could give the direction in two different ways (and both are correct): either in reference to how far north or south of the latitude line, or how far east or west of the longitude line.

Ex: I could describe where Polson is from Helena in both of the following ways:

130 miles and 40° north of west of Helena **or** 130 miles and 50° west of north of Helena

Provide two different descriptions (for each) of where the following are in reference to Helena.

Dillon, Montana:

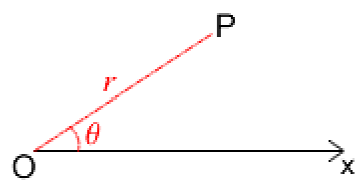
1. 100 miles and 70° south of west of Helena
2. 100 miles and 20° west of south of Helena

Kalispell, Montana:

1. 150 miles and 50° north of west
2. 150 miles and 40° west of north

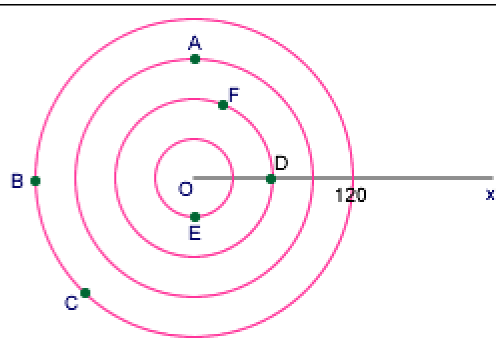
The use of a distance and direction as a means of describing position is far more natural than using two distances on a grid. This means of location is used in many applications (animation, aviation, computer graphics, construction, engineering, and the military) and in math we call it polar coordinates.

The polar coordinates of a point describe its position in terms of a distance from a fixed point (the origin) and an angle measured from a fixed direction which is the angle made by the x-axis, the origin, and the point, measured in a counterclockwise direction.



We say that (r,θ) are the polar coordinates of the point P, where r is the distance P is from the origin O and θ the angle made by $\angle POx$.

Here are some points on a plane and a list of five sets of polar coordinates. Match each letter with its polar coordinates given.

	$(60, 0)$ D $(30, 270)$ E $(120, 225)$ C $(90, 90)$ A $(60, 60)$ F $(120, 180)$ B

So far, I have measured the angles in degrees (°) but the normal convention is to use radians. Recall there are 2π radians in a full turn (360°) and therefore π radians in a half turn (180°).

Looking at the point (90,90) in our list above, note it would be (90, π/2) if the angle is measured in radians. Convert all the following polar coordinates so their angle is measured in radians.

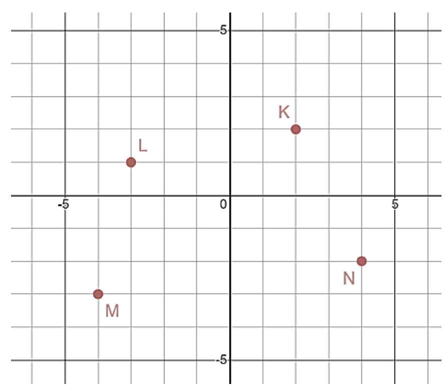
$$(60, 0) \quad (60, 0)$$
$$(30, 270) \quad (30, 3\pi/2)$$
$$(120, 225) \quad (120, 5\pi/4)$$
$$(90, 90) \quad (90, \pi/2)$$
$$(60, 60) \quad (60, \pi/3)$$
$$(120, 180) \quad (120, \pi)$$

Note: There are in fact an infinite number of ways you can write any point using polar coordinates because you can always add 2π, or 4π, or 6π ... to the angle and still end up pointing in the same direction! In the example above the general coordinates for A would be (90, 2nπ+π/2), where n is an integer.

This also means that the polar coordinates of the pole O are (0,θ) where θ can be any angle.

The Relationship Between Polar and Cartesian Coordinates

For the graph and points given (K, L, M, and N) list their cartesian coordinates (I have done K to start).



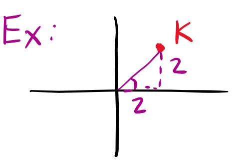
K : (2, 2)	L: (-3, 1)
M: (-4, -3)	N: (4, -2)

See if you can figure out how to list their polar coordinates. Note, do one where you list the polar coordinates with degrees (rounded to nearest tenth when necessary) first and a second where you list the polar coordinates with the angle measured in radians.

Note: when listing polar coordinates it is (r, θ) where *r* is the straight line distance from the origin and θ is the angle made between x-axis, the origin, and the point (measured in a counterclockwise fashion).

Polar Coordinates with Degrees	
K : (2.8, 45)	L: (3.2, 161.6)
M: (5, 216.9)	N: (4.5, 333.4)

Ex:



$\tan\theta = \frac{2}{2}$

$\tan^{-1}\left(\frac{2}{2}\right) = \theta$

$= 45°$

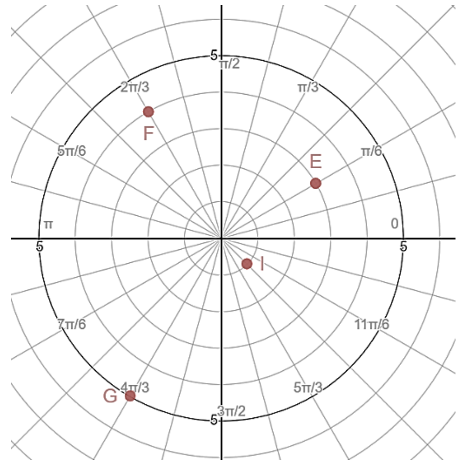
Polar Coordinates with Radians	
K : (2.8, π/4)	L: (3.2, 2.8)
M: (5, 3.8)	N: (4.5, 5.8)

Ex: $161.6° \times \frac{\pi}{180°} = 2.8$ radians

Now, let's see if you can work it backwards!

The following points are given on what is called a polar grid (also called a polar graph or polar coordinate system).

For the graph and points given (E, F, G, and I) list their polar coordinates (E has already been done for you to start).

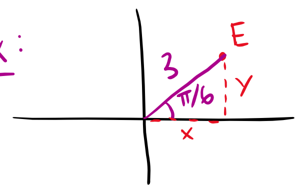


E : $(3, \frac{\pi}{6})$	F: $(4, 2\pi/3)$
G: $(5, 4\pi/3)$	I: $(1, 7\pi/4)$

See if you can figure out how to list their cartesian coordinates.

Cartesian Coordinates	
E : $(2.6, 1.5)$	F: $(-2, 3.5)$
G: $(-2.5, -4.3)$	I: $(0.7, -0.7)$

Ex:



$\cos\left(\frac{\pi}{6}\right) = \frac{x}{3} \Rightarrow x = 3\cos\left(\frac{\pi}{6}\right) = 2.6$

$\sin\left(\frac{\pi}{6}\right) = \frac{y}{3} \Rightarrow y = 3\sin\left(\frac{\pi}{6}\right) = 1.5$

<u>**Summarizing Your Findings**</u>

Now that you have worked a few, see if you can figure out the general formula that will allow you to determine the cartesian x and y-coordinates for a point if you are given the polar coordinates (radius and angle) first.

$x = r \cdot \cos\theta$

$y = r \cdot \sin\theta$

Can you do it backwards? See if you can figure out the general formula that will allow you to determine the polar coordinates (radius and angle) if you are given the cartesian x and y-coordinates first.

$r = \sqrt{x^2 + y^2}$

$\theta = \tan^{-1}\left(\frac{y}{x}\right) \leftarrow$ note: this only gives the reference angle. The user needs to determine the final value based on the quadrant

If you get stuck on this last part, watch the following video: https://tinyurl.com/PolarGraphHelp