Graphing Rational Functions Exploration Name: ______________________________ Per: ______

$$f(x) = \frac{5}{x-1}$$

Let's start by making a table:

X	Y
-2	
-1	
0	
1	
2	

1) Are there any x-values that we cannot use?

2) What is the domain, considering your answer to the previous question?

3) Is there an issue? Describe what that might be using your own words:

4) What is causing this issue? Could we have seen this from the equation itself before making the table?

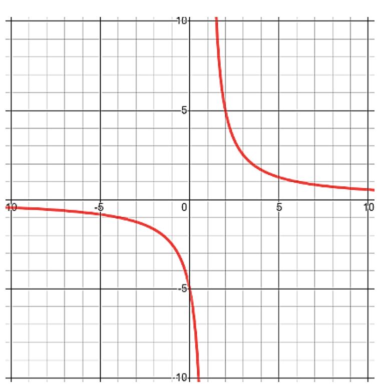
5) Let's look closer at the interesting part – what is happening around x = 1?

X	Y
0.5	
0.9	
0.99	
0.999	
1	
1.001	
1.01	
1.1	
1.5	

6) What is happening on the LEFT side of x = 1?

7) What is happening on the RIGHT side of x = 1?

8) How are these similar? What is the difference?



9) Looking at the graph of this function, what is the y-value when x = 1?

10) Is there a way to identify this same 'issue' by looking at the graph alone?

Graphing Rational Functions Exploration

When we have **excluded values** in a function, we have a **restricted domain**. These impossible x-values cause **vertical asymptotes** where the y-values of the function approach either positive or negative infinity.

Now let's take a look at the following function: $f(x) = \frac{3}{x^2+1}$

Start by making a table:

X	Y
-50	
-10	
0	
10	
50	

11) Are there any x-values that we cannot use?

12) What is the domain?

Let's look closer at the interesting part – what is happening as *x* approaches positive and negative infinity? (fill in the table to see some larger values that are closer to positive or negative infinity)

X	Y
-1000	
-500	
-100	
-10	
0	
10	
100	
500	
1000	

13) As x approaches positive and negative infinity, what values of *y* does the function approach?

This value that *y* approaches as *x* gets really large (either positively or negatively) is the **horizontal asymptote**. Just like the vertical asymptote, this is a value that the function approaches, but never reaches in the long run.

$$f(x) = \frac{3x^2}{x+1}$$

Let's start by making a table:

X	Y
-5	
-2	
-1	
0	
5	

14) Are there any x-values that we cannot use?

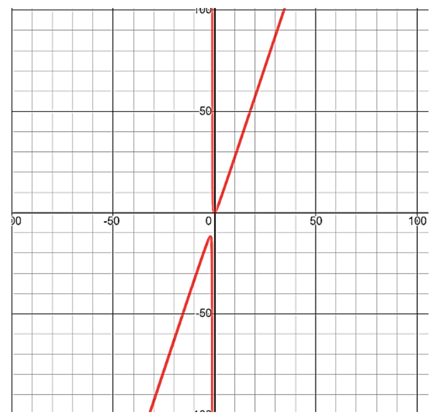
15) What is the domain?

Graphing Rational Functions Exploration

Let's look closer at the interesting part – what is happening as *x* approaches positive and negative infinity? (fill in the table to see some larger values that are closer to positive or negative infinity)

X	Y
-1000	
-100	
-2	
-1.5	
-1	
-0.5	
0	
100	
1000	
10000	

16) As x approaches positive and negative infinity, what values of y does the function approach?



This function has what's called a **slant asymptote** – a line that the function approaches. You can see this in the graph to the right.

In order to find the slant asymptote, we must use polynomial or synthetic division on the original function. The quotient (without the remainder) is the equation for the slant asymptote.

We will study this in a later lesson.

Let's look at our final function.

$$f(x) = \frac{3x+2}{5x+1}$$

Let's start by making a table:

X	Y
-10	
-5	
-0.2	
0	
5	

17) Are there any x-values that we cannot use?

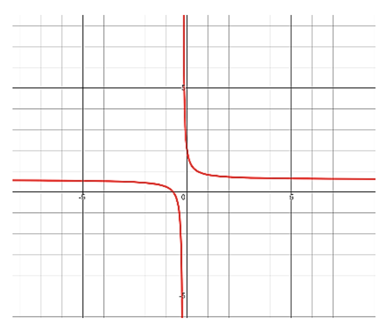
18) What is the domain?

Let's look at the end behavior – what happens as *x* approaches positive or negative infinity?

X	Y
100000	
1000	
10	
0	
-0.2	
-10	
-1000	
-10000	
-100000	

19) As x approaches positive and negative infinity, what values of y does it approach?

20) What is the horizontal asymptote of this function?



Graphing Rational Functions Exploration

Now that we've seen some examples, let's write a rule that can be used to describe where the vertical and horizontal or slant asymptotes will be!

Let's start with the vertical:

The vertical asymptote of a rational function will be

With respect to the horizontal asymptote, there are three possible cases: the degree of the top is bigger, the degree of the top is smaller, or the degrees of the top and bottom are the same. Looking back at the examples we've just completed, try to finish the following statements in your own words. The result should be a rule you can use to always know whether you have a horizontal or slant asymptote, and where it goes!

If the degree of the numerator is greater than the degree of the denominator, then:

If the degree of the numerator is smaller than the degree of the denominator, then:

If the degrees of the numerator and denominator are the same, then: