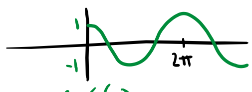
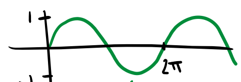
Honors Precalculus

Name:________________________________ Per: _____

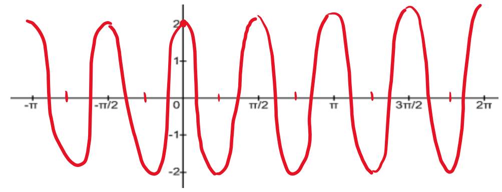
Semester 2 PRACTICE TEST *Use an extra sheet to show work if you run out of room, or the blank last back page.*

1. Identify the amplitude and period of each.

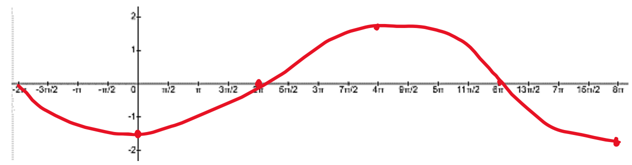
$2cos\,(4x)$	$-1.5sin\,(x/4)$
Amplitude: $\lvert 2 \rvert = \boxed{2}$	Amplitude: $\lvert -1.5 \rvert = \boxed{1.5}$
Period: $\frac{2\pi}{b} = \frac{2\pi}{4} = \boxed{\frac{\pi}{2}}$	Period: $\frac{2\pi}{1/4} = \boxed{8\pi}$
	

2. Sketch the graph for each from Problem 2.

$2cos\,(4x)$



$-1.5\sin\left(\frac{x}{4}\right)$



3.

Identify the information for $f(x) = 3\cos\left(\frac{1}{4}\left(x - \frac{\pi}{2}\right)\right) + 5$	Identify the information for $f(x) = 0.5\sin(2x + \pi) + 1$
	$f(x) = 0.5\sin(2(x + \pi/2)) + 1$
Horizontal shift: $\boxed{\text{Right } \frac{\pi}{2}}$	Horizontal shift: $\boxed{\text{Left } \pi/2}$
Period: $\frac{2\pi}{b} = \frac{2\pi}{1/4} = \boxed{8\pi}$	Period: $\frac{2\pi}{b} = \frac{2\pi}{2} = \boxed{\pi}$
Amplitude: $\lvert 3 \rvert = \boxed{3}$	Amplitude: $\lvert 0.5 \rvert = \boxed{0.5}$
Vertical shift: $\boxed{\text{Up } 5}$	Vertical shift: $\boxed{\text{Up } 1}$

4. Use the parent table for $\sin x$ to fill in the table for $f(x) = 0.5\sin(2x + \pi) + 1$

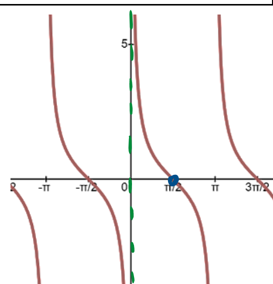
$+\pi$

x	$-\frac{\pi}{2}$	$-\frac{\pi}{4}$	0	$\frac{\pi}{4}$	$\frac{\pi}{2}$
$f(x)$	1	1.5	1	0.5	1

x	0	π/2	π	3π/2	2π
sin(x)	0	1	0	-1	0

×0.5 and then add 1

5. Use the fact that $f(x) = \cot x = \frac{\cos x}{\sin x}$ to explain why the graph has a vertical asymptotes at $x = 0$ and a zero at $x = \frac{\pi}{2}$.



$\cot(x) = \frac{\cos x}{\sin x}$

There is a V.A. @ $x=0$ because that is where $\sin(x)=0$, so that is where we would be dividing by 0.

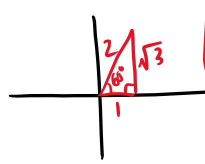
$\cot(x) = \frac{\cos x}{\sin x} = 0$

$\cot(x)$ has a zero (x-int) at $x = \pi/2$ because $\cos(x) = 0$ when $x = \pi/2$.

6. Find the exact value of each in radians. Sketch a picture if necessary.

$\arccos\left(\frac{1}{2}\right)$

$\cos^{-1}\left(\frac{1}{2}\right) = \text{angle}$

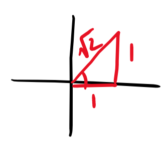


$\boxed{\frac{\pi}{3}}$

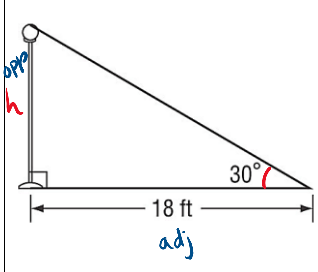
$\tan^{-1}(1)$

$\tan^{-1}(1) = \text{angle}$

$\boxed{\frac{\pi}{4}}$



7. Find the height of the lamppost to the nearest tenth of a foot.

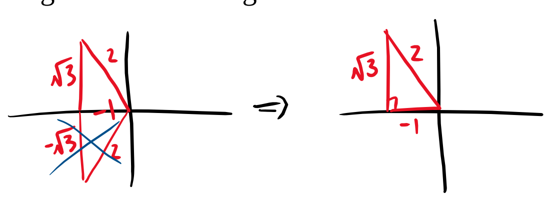


$\tan(30) = \frac{h}{18}$

$h = 18 \cdot \tan(30)$

$\boxed{h = 10.4 \text{ ft}}$

8. Use the fact that $\cos x = -1/2$ and $\tan x < 0$ to get the other 5 trig values.



$\sin(x) = \frac{\sqrt{3}}{2}$ $\cos(x) = -\frac{1}{2}$ $\tan(x) = \frac{\sqrt{3}}{-1} = -\sqrt{3}$

$\csc(x) = \frac{2}{\sqrt{3}}$ or $\frac{2\sqrt{3}}{3}$ $\sec(x) = -2$ $\cot(x) = -\frac{1}{\sqrt{3}}$ or $-\frac{\sqrt{3}}{3}$

9. Verify the following trig identities.

a) $\sin t \cdot \csc t = 1$

$\cancel{\sin t} \cdot \frac{1}{\cancel{\sin t}} = 1$

$1 = 1$ ✓

b) $\frac{\cot x \cdot \tan x}{\cos x} = \sec x$

$\frac{\frac{1}{\cancel{\tan x}} \cdot \cancel{\tan x}\ 1}{\cos x} = \sec x \nearrow \frac{1}{\cos x} = \sec x$

$\sec x = \sec x$ ✓

c) $(1 + \sin x)(1 - \sin x) = \cos^2 x$

$1 \cancel{- \sin x} \cancel{+ \sin x} - \sin^2 x = \cos^2 x$

$1 - \sin^2 x = \cos^2 x$

$\cos^2 x = \cos^2 x$ ✓

$\sin^2 x + \cos^2 x = 1$

$\cos^2 x = 1 - \sin^2 x$

Pythagorean Identity

d) $\cos x\,(\tan^2 x + 1) = \sec x$

$1 + \tan^2 x = \sec^2 x$

$\cos x\,(\sec^2 x) = \sec x$

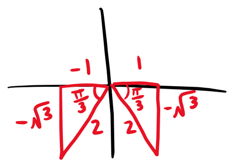
$\cancel{\cos x}\left(\frac{1}{\cos^{\cancel{2}} x}\right) = \sec x \Rightarrow \frac{1}{\cos x} = \sec x$

$\sec x = \sec x$ ✓

10. Solve $2\sin x + \sqrt{3} = 0$ on the interval $[0, 2\pi)$.

$\sin x = \frac{-\sqrt{3}}{2}$

$\sin^{-1}\left(\frac{-\sqrt{3}}{2}\right) = x$



$x = \pi + \frac{\pi}{3}$ $\quad$ $x = 2\pi - \frac{\pi}{3}$

$\boxed{x = \frac{4\pi}{3}}$ $\quad$ $\boxed{x = \frac{5\pi}{3}}$

11. Use a sum and difference formula to find $\sin(15°)$ exactly (no decimals).

$\sin(60 - 45) = \sin 60 \cos 45 - \cos 60 \sin 45$

$\left(\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{2}}{2}\right) - \left(\frac{1}{2}\right)\left(\frac{\sqrt{2}}{2}\right)$

$\boxed{\frac{\sqrt{6} - \sqrt{2}}{4}}$

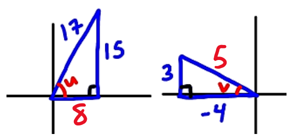
$\left(\frac{\sqrt{3}}{2}\right)\left(\frac{1}{\sqrt{2}}\right) - \left(\frac{1}{2}\right)\left(\frac{1}{\sqrt{2}}\right)$

$\frac{\sqrt{3}}{2\sqrt{2}} - \frac{1}{2\sqrt{2}} = \boxed{\frac{\sqrt{3} - 1}{2\sqrt{2}}}$

12. Given the angles shown, find $\cos(u + v)$

$\sin u = \frac{15}{17}$ $\quad$ $\sin v = 3/5$

$\cos u = 8/17$ $\quad$ $\cos v = -4/5$



$\cos(u+v) = \cos u \cos v - \sin u \sin v$

$= \left(\frac{8}{17}\right)\left(\frac{-4}{5}\right) - \left(\frac{15}{17}\right)\left(\frac{3}{5}\right)$

$= \frac{-32}{85} - \frac{45}{85} = \boxed{\frac{-77}{85}}$

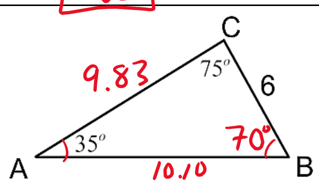
13. Use the figure from Problem 12 to find $\sin(2u)$.

$\sin(2u) = 2\sin u \cos u$

$= 2\left(\frac{15}{17}\right)\left(\frac{8}{17}\right)$

$= \boxed{\frac{240}{289}}$

14. Solve the triangle.



$\frac{c}{\sin(75)} = \frac{6}{\sin(35)}$

$c = \frac{6\sin(75)}{\sin(35)} = 10.10$

$\angle B = 180 - 35 - 75$

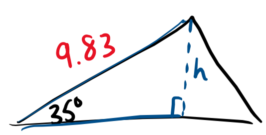
$\angle B = 70°$

$\frac{b}{\sin(70)} = \frac{6}{\sin(35)}$

$b = \frac{6\sin(70)}{\sin(35)} = 9.83$

$b =$ 9.83 $\quad c =$ 10.10 $\quad \angle B =$ 70°

15. Find the area of the triangle in problem 14 by first finding the height.



$\sin(35) = \frac{h}{9.83}$

$h = 9.83 \cdot \sin(35) = 5.64$

$A = \frac{1}{2}bh = \frac{1}{2}(10.10)(5.64) = \boxed{28.48 \text{ units}^2}$

16. Find the area of the triangle in problem 14 by using Heron's formula along with your prior results.

$A = \sqrt{s(s-a)(s-b)(s-c)}$ where $s = \frac{a+b+c}{2}$

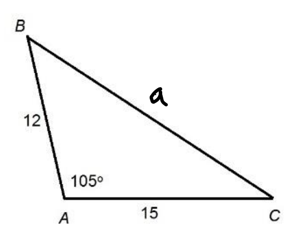
$A = \sqrt{12.97(12.97-6)(12.97-9.83)(12.97-10.1)}$ $\quad$ $s = \frac{6 + 9.83 + 10.1}{2}$

$s = 12.97$

$\boxed{A = 28.54 \text{ units}^2}$

* Differs from #15 simply due to rounding

17. Find the length of side A.



$a^2 = b^2 + c^2 - 2bc\cos A$

$a^2 = 15^2 + 12^2 - 2(15)(12)\cos(105)$

$a^2 = 462.175$

$\boxed{a = 21.5}$

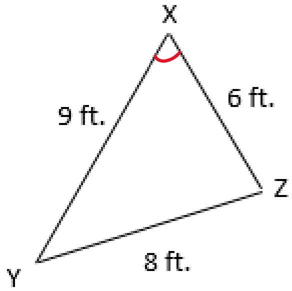
18. Find the measure of angle X.

$\cos X = \frac{y^2+z^2-x^2}{2yz}$

$\cos X = \frac{6^2+9^2-8^2}{2(6)(9)}$

$\cos X = \frac{53}{108}$

$\cos^{-1}\left(\frac{53}{108}\right) = X$



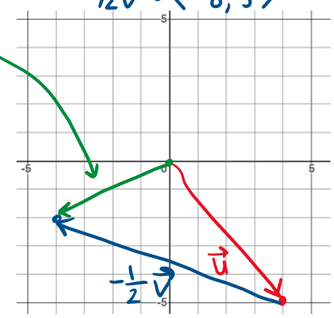
$\boxed{X = 60.6^\circ}$

19. Given $\vec{u} = <4, -5>$ and $\vec{v} = <16, -6>$

$-\frac{1}{2}\vec{v} = <-8, 3>$

a) sketch $\boxed{\vec{u} - \frac{1}{2}\vec{v}}$.

b) find $\vec{u} - \frac{1}{2}\vec{v}$ algebraically.



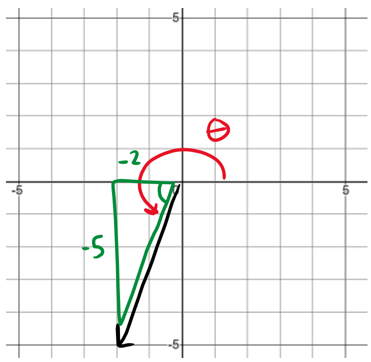
$\vec{u} - \frac{1}{2}\vec{v}$

$<4, -5> - \frac{1}{2}<16, -6>$

$<4, -5> - <8, -3> = \boxed{<-4, -2>}$

20. Sketch in vector $\boldsymbol{v} = -2\boldsymbol{i} - 5\boldsymbol{j}$ and then find the magnitude and direction of $\boldsymbol{v}$.

$\vec{v} = <-2, -5>$



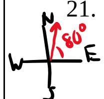
$\|\vec{v}\| = \sqrt{v_x^2 + v_y^2} = \sqrt{(-2)^2 + (-5)^2} = \sqrt{29}$

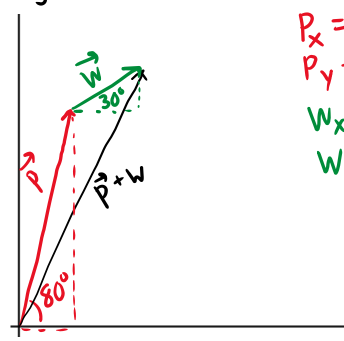
$\boxed{\|v\| = \sqrt{29} \approx 5.39}$

$\tan^{-1}\left(\frac{-5}{-2}\right) = 68.2^\circ$

$\theta = 180^\circ + 68.2^\circ = \boxed{248.2^\circ}$

21. A plane with an airspeed of 350 mph at a bearing of E 80° N encounters wind with a velocity of 60 mph at E 30° N. Find the resultant (airplane + wind) speed and direction of the two.





$P_x = 350 \cos 80 = 60.8$

$P_y = 350 \sin 80 = 344.7$

$W_x = 60 \cos 30 = 52$

$W_y = 60 \sin 30 = 30$

$\vec{p} = <60.8, 344.7>$

$+ \vec{w} = <52, 30>$

$\vec{p} + \vec{w} = <112.8, 374.7>$

$\|\vec{p} + \vec{w}\| = \sqrt{(112.8)^2 + (374.7)^2} = \boxed{391.3 \text{ mph}}$

$\theta = \tan^{-1}\left(\frac{374.7}{112.8}\right) = 73.2^\circ$ $\boxed{E\ 73.2^\circ\ N}$

22. Graph the equation $(x-4)^2 + (y+3)^2 = 25$.

Circle:

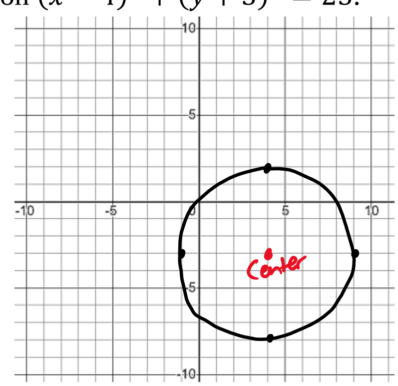
$(x-h)^2 + (y-k)^2 = r^2$

center: (h, k)

radius $= r$

Center: $(4, -3)$

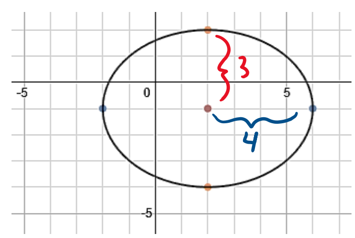
radius $= \sqrt{25} = 5$



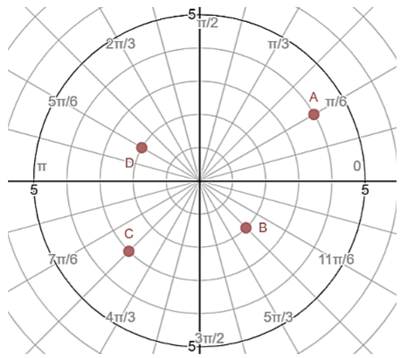
23. Write the equation for the ellipse shown.

Center: $(2, -1)$

$\boxed{\frac{(x-2)^2}{16} + \frac{(y+1)^2}{9} = 1}$



24. List the polar coordinates for the points shown.



A: $(4, \pi/6)$

B: $(2, 7\pi/4)$

C: $(3, 5\pi/4)$

D: $(2, 5\pi/6)$

25. a) Convert point D from problem 24 to rectangular coordinates. D: $(2, 5\pi/6)$

$x = 2\cos(5\pi/6) = -1.73$

$y = 2\sin(5\pi/6) = 1$

$\boxed{D: (-1.73, 1)}$

b) Convert (4, -4) to polar coordinates. $\boxed{(4\sqrt{2}, 7\pi/4)}$ or $\boxed{(5.66, -\pi/4)}$

$r = \sqrt{x^2+y^2} = \sqrt{4^2+(-4)^2} = \sqrt{32} = \sqrt{16}\sqrt{2} = 4\sqrt{2} = 5.66$

4, -4, θ

$\theta = \tan^{-1}\left(\frac{-4}{4}\right) = \frac{-\pi}{4}$ or $\frac{7\pi}{4}$

26. Use back substitution to solve.

$x - y + 2z = 22$ $17+11-6=22$ ✓

✓ $-33-8(-3)=-9$ $3y - 8z = -9$

$z = -3$ ✓

$3y - 8(-3) = -9$

$3y + 24 = -9$

$3y = -33$

$y = -11$

$x-(-11)+2(-3)=22$

$x+11-6=22$

$x+5=22$

$x=17$

$\boxed{(17, -11, -3)}$

27. Get the following to Row Echelon form.

$4x + y - 3z = -11$

$2x - 3y + 2z = 9$

$x + y + z = 3$

① $\boxed{R_1 - 2R_2 \to R_2}$

$4x+y-3z=-11$

$7y-7z=-29$

$x+y+z=3$

② $\boxed{R_1 - 4R_3 \to R_3}$

$4x+y-3z=-11$

$7y-7z=-29$ $21y-21z=-87$

$-3y-7z=-23$ $-21y-49z=-161$

③ $\boxed{3R_2 + 7R_3 \to R_3}$

$4x+y-3z=-11$

$7y-7z=-29$

$-70z=-248$

④ $\boxed{-\frac{1}{70}R_3 \to R_3}$

$$\boxed{\begin{aligned} 4x+y-3z&=-11\\ 7y-7z&=-29\\ z&=3.543 \end{aligned}}$$

28. Write the following as an augmented matrix and then use a calculator to get it to RREF to solve it.

$$A = \begin{bmatrix} 4x + y - 3z = -11 \\ 2x - 3y + 2z = 9 \\ x + y + z = 3 \end{bmatrix}$$

$$\text{rref}(A) = \begin{bmatrix} 1 & 0 & 0 & 0.057 \\ 0 & 1 & 0 & -0.6 \\ 0 & 0 & 1 & 3.543 \end{bmatrix}$$

$\boxed{(0.057, -0.6, 3.543)}$

29. Let A and B be the matrices shown. Find the following:

$A = \begin{bmatrix} -5 & 4 \\ 2 & -9 \end{bmatrix}$ 2×2 $B = \begin{bmatrix} 8 & -2 \\ 4 & 0 \end{bmatrix}$ 2×2

$3A - 4B$

$$\begin{bmatrix} -15 & 12 \\ 6 & -27 \end{bmatrix} - \begin{bmatrix} 32 & -8 \\ 16 & 0 \end{bmatrix}$$

$$\boxed{\begin{bmatrix} -17 & 20 \\ -10 & -27 \end{bmatrix}}$$

$A \cdot B$

$$\begin{bmatrix} -5(8)+4(4) & -5(-2)+4(0) \\ 2(8)+(-9)(4) & 2(-2)+-9(0) \end{bmatrix}$$

$$\boxed{\begin{bmatrix} -24 & 10 \\ -20 & -4 \end{bmatrix}}$$

30. Find the multiplicative inverse for matrix B from the prior problem.

$$B = \begin{bmatrix} 8 & -2 \\ 4 & 0 \end{bmatrix}$$
(a, b, c, d)

$$B^{-1} = \frac{1}{ad-bc}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix} = \frac{1}{0-(-8)}\begin{bmatrix} 0 & 2 \\ -4 & 8 \end{bmatrix}$$

$$B^{-1} = \boxed{\begin{bmatrix} 0 & 1/4 \\ -1/2 & 1 \end{bmatrix}}$$

31. In trying to solve the system using inverse matrices, a student writes the following. Explain and then correct the error.

$-x + 4y = 8$

$2x - 7y = -5$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 8 \\ -5 \end{bmatrix}\begin{bmatrix} -1 & 4 \\ 2 & -7 \end{bmatrix}^{-1}$$

They multiplied on the wrong side

$$\begin{bmatrix} -1 & 4 \\ 2 & -7 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 8 \\ -5 \end{bmatrix}$$

$A^{-1}\ A \cdot X = A^{-1}B$

$$\boxed{\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -1 & 4 \\ 2 & -7 \end{bmatrix}^{-1}\begin{bmatrix} 8 \\ -5 \end{bmatrix}}$$

32. Fill in the table below and then state the limit.

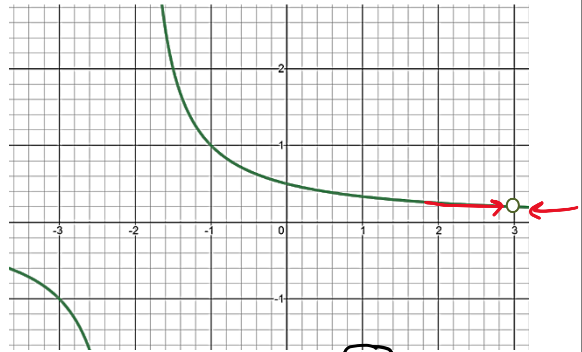
$$f(x) = \frac{x-3}{x^2-x-6}$$

x	2.9	2.95	2.99	3	3.01	3.05	3.1
$f(x)$	0.2041	0.2020	0.2004		0.1996	0.198	0.1961

*Go out to 4 decimal places.

$$\lim_{x\to 3}\frac{x-3}{x^2-x-6} = \boxed{0.2} = \boxed{1/5}$$

33. State the limit using the graph.



$$\lim_{x\to 3} f(x) = \boxed{\frac{1}{5}} = 0.2$$

34. Let $f(x) = \frac{x-3}{x^2-x-6}$

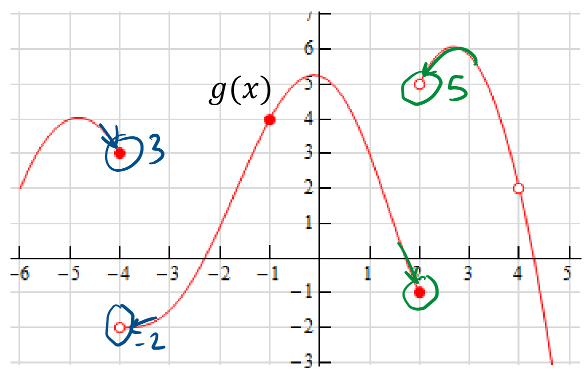
a) State what $f(3)$ is.

$$f(3) = \frac{0}{9-3-6} = \frac{0}{0}\ \boxed{\text{undefined}}$$

b) Simplify to determine $\lim_{x\to 3}\frac{x-3}{x^2-x-6}$.

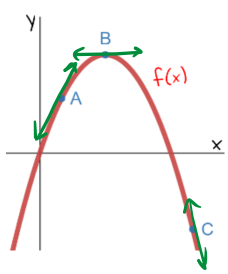
$$\lim_{x\to 3}\frac{\cancel{(x-3)}}{\cancel{(x-3)}(x+2)} = \lim_{x\to 3}\frac{1}{x+2} = \boxed{\frac{1}{5}}$$

35. Use the graph below to answer the following.



$\lim_{x\to -4^-} g(x) = 3$	$\lim_{x\to -4} g(x) =$ DNE
$\lim_{x\to 2^+} g(x) = 5$	$\lim_{x\to 2^-} g(x) = -1$

36. Given the function, state whether the following derivative values would be positive, negative, or zero.



$f'(A)$ positive

$f'(B)$ zero

$f'(C)$ negative

37. Use the limit process to find the derivative of $f(x) = -x^2 + 3x$.

$(x, f(x)) \quad (x+h, f(x+h))$

$$f'(x) = \lim_{h\to 0}\frac{f(x+h) - f(x)}{h}$$

$$f'(x) = \lim_{h\to 0}\frac{[-(x+h)^2 + 3(x+h)] - [-x^2 + 3x]}{h}$$

$$f'(x) = \lim_{h\to 0}\frac{\cancel{-x^2} - 2xh - h^2 + \cancel{3x} + 3h + \cancel{x^2} - \cancel{3x}}{h}$$

$$f'(x) = \lim_{h\to 0}\frac{\cancel{h}(-2x - h + 3)}{\cancel{h}}$$

$$f'(x) = \lim_{h\to 0}(-2x - \cancel{h} + 3)$$ 0

$$\boxed{f'(x) = -2x + 3}$$

38. Use your result from #37 to find $f'(1.5)$ and interpret what that means (note, the graph on #36 is the graph of $f(x) = -x^2 + 3x$).

$f'(1.5) = -2(1.5) + 3$

$f'(1.5) = 0$

x-coordinates

slope of the original function at $x = 1.5$

At $x = 1.5$ on the original graph, the slope at $x = 1.5$ is 0, thus the graph goes flat there. Based on the picture we now know the vertex's x-coordinate is 1.5.