Honors Precalculus

Name:________________________________ Per: ____

Semester 1 PRACTICE TEST

Use an extra sheet to show work if you run out of room.

1. Given $f(x)$ is linear, $f(\overset{x}{2}) = \overset{y}{6}$, and the graph for $f(x)$ is perpendicular to $g(x) = \frac{1}{4}x - 7$, find the equation for $f(x)$.

$f(x) = mx + b$

$f(x) = -4x + b$

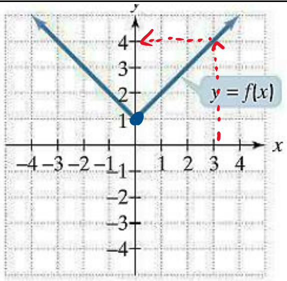
$6 = -4(2) + b$

$6 = -8 + b$

$14 = b$

$\boxed{f(x) = -4x + 14}$

2. Given the graph $f(x)$, fill in the blanks.



a) $f(\overset{x}{3}) = $ 4

b) $f($ 0 $) = \overset{y}{1}$

3. State the domain and range for each.

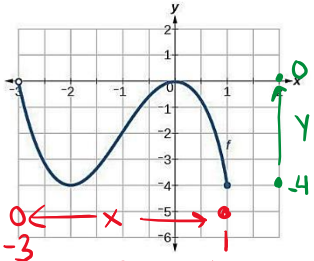
$h(x) = \sqrt{x - 5}$

$x - 5 \geq 0$

$x \geq 5$

Domain: $x \geq 5$

Range: $y \geq 0$



Domain: $-3 < x \leq 1$

$(-3, 1]$

Range: $-4 \leq y \leq 0$

$[-4, 0]$

4. Graph.

$$y = \begin{cases} \boxed{\frac{1}{2}x + 4}, & \underline{x \leq -2} \\ \boxed{-\frac{1}{3}x - 5}, & \underline{x > -2} \end{cases}$$

Simplify the complex expressions.

5. Given the table for $f(x)$, fill in the table for $g(x)$ if $g(x) = \underline{2}f(x - 4) + 5$.

Right 4

x	0	1	2
f(x)	3	7	-5

add 4

multiply by 2 and then add 5

x	4	5	6
g(x)	11	19	-5

6. Let $f(x) = x^2 + 3x$ and $g(x) = -2x - 6$. Find:

$f - g$

$(x^2 + 3x) - (-2x - 6)$

$x^2 + 3x + 2x + 6$

$\boxed{x^2 + 5x + 6}$

f/g

$$\frac{x^2 + 3x}{-2x - 6} = \frac{x\cancel{(x+3)}}{-2\cancel{(x+3)}}$$

$$\boxed{\frac{x}{-2} = -\frac{x}{2}}$$

fg

$(x^2 + 3x)(-2x - 6)$

$-2x^3 - 6x^2 - 6x^2 - 18x$

$\boxed{-2x^3 - 12x^2 - 18x}$

7. Let $f(x) = x^2 + 5x$ and $g(x) = x - 4$. Find:

$f(g(1))$

$g(1) = 1 - 4 = \underline{-3}$

$f(g(1)) = f(-3)$

$= (-3)^2 + 5(-3)$

$= 9 - 15 = \boxed{-6}$

$g(f(1))$

$f(1) = 1^2 + 5(1) = 6$

$g(f(1)) = g(6) = 6 - 4$

$= \boxed{2}$

$f(g(x))$

$f(x - 4)$

$=$

$(x - 4)^2 + 5(x - 4)$

$(x - 4)(x - 4) + 5x - 20$

$x^2 - 8x + 16 + 5x - 20$

$\boxed{x^2 - 3x - 4}$

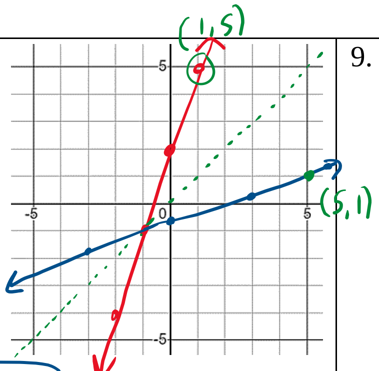
8. Find the inverse function of $f(x) = 3x + 2$ and graph both.

$y = 3x + 2$

$\frac{y}{3} - \frac{2}{3} = \frac{3x}{3}$

$\frac{1}{3}y - \frac{2}{3} = x$

$\boxed{f^{-1}(x) = \frac{1}{3}x - \frac{2}{3}}$



9. Write the equation, in vertex form $f(x) = a(x-h)^2 + k$, for the parabola that has a vertex of (-6, 2) and a y-intercept of 14.

$(0,14)$
x y

$f(x) = a(x+6)^2 + 2$

$14 = a(0+6)^2 + 2$

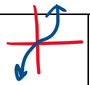
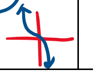
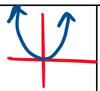
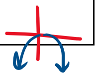
$14 = 36a + 2$

$12 = 36a$

$\frac{1}{3} = a$

$\boxed{f(x) = \frac{1}{3}(x+6)^2 + 2}$

10. Circle the correct end-behavior for each.

$f(x) = x^3 - 4x^2$ [✓ a) ↙ ↗] b) ↖ ↗ c) ↖ ↘ d) ↙ ↘ 	$f(x) = -x^4(x+2)$ $-x^5$ a) ↙ ↗ b) ↖ ↗ [✓ c) ↖ ↘] d) ↙ ↘ 	$f(x) = x^2 + 4$ a) ↙ ↗ [✓ b) ↖ ↗] c) ↖ ↘ d) ↙ ↘ 	$-x^3 \cdot x^3$ $f(x) = -x^3(x-1)^3$ $-x^6$ a) ↙ ↗ b) ↖ ↗ c) ↖ ↘ [✓ d) ↙ ↘] 

11. Sketch the graph of the polynomial and clearly mark key points.

$-x^2 \cdot x \to -x^3$

$y = -1.5(x+1)^2(x-2)$

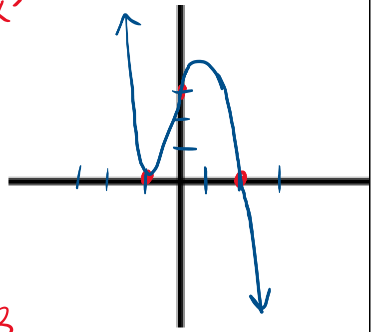
$0 = -1.5(x+1)^2(x-2)$

$x = -1$ bounce

$x = 2$ linear

y -intercept: (0 , 3)

$y = -1.5(1)^2(-2) = 3$



12. Use long or synthetic division to divide and then find all the other factors.

$(2x^3 + x^2 - 25x + 12) \div (x-3)$

$$\begin{array}{r} 2x^2+7x-4 \\ x-3\,\overline{)\,2x^3+x^2-25x+12} \\ -\,(2x^3-6x^2) \\ \hline 7x^2-25x \\ -\,(7x^2-21x) \\ \hline -4x+12 \\ -\,(-4x+12) \\ \hline \end{array}$$

$2x^2 + 7x - 4 = \boxed{(2x-1)(x+4)}$

$$\begin{array}{r|rrrr} 3 & 2 & 1 & -25 & 12 \\ + & & 6 & 21 & -12 \\ \hline & 2 & 7 & -4 & 0 \end{array}$$

$2x^2 + 7x - 4$

$\boxed{(2x-1)(x+4)}$

$x = 1/2$ $x = -4$

13. Use the Rational Root/Zero Test to state all possible roots for $f(x) = x^3 + 3x^2 - 6x - 8$

possible roots $= \frac{\text{factors of } -8}{\text{factors of } 1} = \frac{\pm1, \pm2, \pm4, \pm8}{\pm1} = \boxed{\pm1, \pm2, \pm4, \pm8}$

14. Find one that is actually a root (and how you know) from Problem 13.

$f(1) = 1^3 + 3(1)^2 - 6(1) - 8 = 1 + 3 - 6 - 8 \neq 0$

$f(-1) = (-1)^3 + 3(-1)^2 - 6(-1) - 8 = -1 + 3 + 6 - 8 = 0$

$\boxed{(x+1) \text{ is a factor since } x = -1 \text{ is a root}}$

15. Simplify $(11 - i) - (-2 + 5i)$

$(11 - i) - (-2 + 5i)$

$11 - i + 2 - 5i = \boxed{13 - 6i}$

16. Simplify $\frac{4-5i}{2+3i}$

$i^2 = -1$

$\frac{4-5i}{2+3i} \cdot \frac{(2-3i)}{(2-3i)} = \frac{8 - 12i - 10i + 15i^2}{4 \cancel{-6i+6i} - 9i^2} = \frac{8 - 22i - 15}{13}$

$\frac{-7 - 22i}{13} = \boxed{-\frac{7}{13} - \frac{22}{13}i}$

17. Solve $4x^2 + 10 = -26$ using complex numbers.

$4x^2 = -36$

$\sqrt{x^2} = \sqrt{-9}$

$\boxed{x = \pm 3i}$

18. Find a fourth-degree polynomial that has zeros 2, 3, and -5 (-5 has multiplicity 2) and $f(0) = 50$

$f(x) = a(x-2)(x-3)(x+5)^2$

$50 = a(0-2)(0-3)(0+5)^2$

$\frac{50}{150} = \frac{a(150)}{150}$

$a = \frac{1}{3}$

$\boxed{f(x) = \frac{1}{3}(x-2)(x-3)(x+5)^2}$

19. State the vertical and horizontal asymptote(s) of each function.

$y = \frac{-2}{x+6}$

VA: $x+6=0$ $\boxed{x=-6}$

$\frac{-2}{x}$

HA: as $x \to \infty, y \to 0$ $\boxed{y=0}$

$\frac{-2}{\infty}$

$y = \frac{2x^2}{x^2+7x+12}$

VA: $x^2+7x+12=0$ $(x+3)(x+4)=0$ $\boxed{x=-3}$ $\boxed{x=-4}$

HA: $\frac{2\cancel{x^2}}{\cancel{x^2}} = 2$ as $x \to \infty, y \to 2$ $\boxed{y=2}$

20. Put all this together and sketch a graph of the rational function $f(x) = \frac{x-1}{x^2+x-6}$

HA: $\frac{x}{x^2} = \frac{1}{x}$ as $x \to \infty, f(x) \to 0$ $\boxed{y=0}$

VA: $x^2+x-6=0$

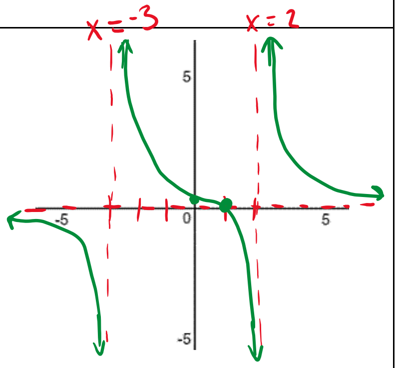
$(x+3)(x-2)=0$

$\boxed{x=-3}$ $\boxed{x=2}$

Zero(s): $0 = \frac{x-1}{x^2+x-6}$

$x-1=0$ $\boxed{x=1}$

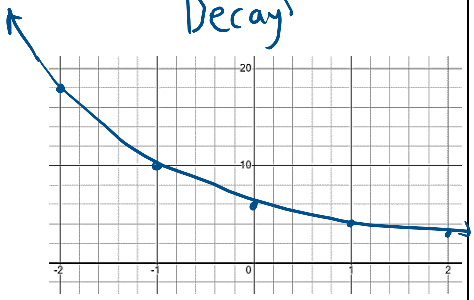
Y-int: $f(0) = \frac{0-1}{0^2+0-6} = \frac{-1}{-6} = \frac{1}{6}$



21. Make a table and graph $f(x) = 4(0.5)^x + 2$

x	f(x)
-2	18
-1	10
0	6
1	4
2	3

Decay



22. Find the value of an investment of $40,000 for 5 years at an interest rate of 3% if the money is compounded:

a) monthly

$A = P(1+\frac{r}{n})^{nt}$

$A = 40{,}000(1+\frac{.03}{12})^{12(5)}$

$\boxed{A = \$46{,}464.67}$

b) continuously

$A = Pe^{rt} = 40{,}000e^{0.03(5)} = \boxed{\$46{,}473.37}$

23. Evaluate each and justify by writing in exponential form.

$ln(\sqrt{e}) =$ ___1/2___ since $e^{1/2} = \sqrt{e}$

$log_{10}(1000) =$ ___3___ since $10^3 = 1000$

$log_{10}\left(\frac{1}{100}\right) =$ ___-2___ since $10^{-2} = \frac{1}{10^2} = \frac{1}{100}$

10^{-2}

$log_4(64) =$ ___3___ since $4^3 = 64$

$\log_b(a) = n \to b^n = a$

24. In 1970, the US population was 203 million. By, 2010 it was 308 million. Use these to find the exponential equation ($A = A_0e^{kt}$) for the US population, in millions, t years after 1970.

$A = 203e^{kt}$

$308 = 203e^{40k}$

$\frac{308}{203} = e^{40k}$

$\ln(308/203) = \ln(e^{40k})$

$\ln(308/203) = 40k \underbrace{\ln(e)}_{=1}$

$K = \frac{\ln(308/203)}{40}$

$K = 0.0104$

$\boxed{A = 203e^{0.0104t}}$

25. Solve for x.

a) $\ln(x-4) - 5 = -3$

$\ln(x-4) = 2 \rightarrow e^2 = x-4$

$e^2 + 4 = x$ ≈ 11.39

b) $16^x - 8 = 56$

$16^x = 64$

$\log(16^x) = \log(64)$

$x \cdot \log(16) = \log(64)$

$x = \frac{\log(64)}{\log(16)} = \boxed{1.5}$

26. How long will it take for the continuous investment in Problem 22 to double?

$\frac{80{,}000}{40{,}000} = \frac{40{,}000e^{0.03t}}{40{,}000}$

$2 = e^{0.03t}$

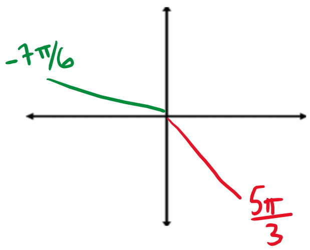
$\ln(2) = 0.03t \underbrace{\ln(e)}_{=1}$

$t = \frac{\ln(2)}{0.03} = 23.1$ yrs

27. Sketch in each of the angles and mark them on the graph.

a) $\frac{5\pi}{3} = \frac{5(180)}{3} = 300$

b) $-\frac{7\pi}{6} = \frac{-7(180)}{6} = -210$

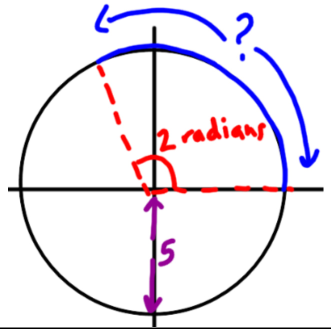


28. If an angle of 2 radians on a circle of radius 5 is shown, what is the length of the arc?

2 radians

⇓

$2(5) = \boxed{10}$



29. Convert between radians and degrees and vice versa.

$\frac{\pi}{8} = \boxed{22.5}°$ $\quad \frac{\pi}{8} \times \frac{180}{\pi} = \frac{180}{8}$

$\frac{13\pi}{12} = 195°$ $\quad \frac{13\cancel{\pi}}{12} \cdot \frac{180°}{\cancel{\pi}} = 195$

$150° = \boxed{\frac{5\pi}{6}}$ $\quad 150° \times \frac{\pi}{180°} = \frac{5\pi}{6}$

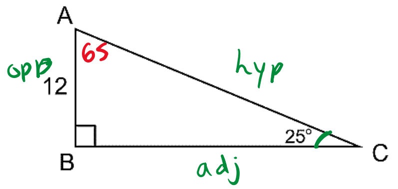
$240° = \boxed{\frac{4\pi}{3}}$ $\quad 240° \times \frac{\pi}{180°} = \frac{4\pi}{3}$

30. For the triangle shown, find the following.

$\tan(25) = \frac{12}{BC}$

$BC = \frac{12}{\tan(25)}$

$BC = 25.73$



$\sin(25) = \frac{12}{AC}$

$AC = 12/\sin(25)$

↓

$m\angle A = 90 - 25$

$m\angle A = 65$ $\quad BC = 25.73$ $\quad AC = 28.39$

31. Use basic trig identities to prove the following.

a) $\cot\theta \cdot \sin\theta = \cos\theta$

$\frac{\cos\theta}{\cancel{\sin\theta}} \cdot \frac{\cancel{\sin\theta}}{1} = \cos\theta$

$\cos\theta = \cos\theta$ ✓

b) $(1 + \cos\theta)(1 - \cos\theta) = \sin^2\theta$

$1 - \cancel{\cos\theta} + \cancel{\cos\theta} - \cos^2\theta = \sin^2\theta$

$1 - \cos^2\theta = \sin^2\theta$

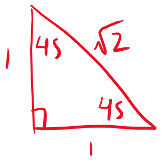
$\boxed{\text{we know } \sin^2\theta + \cos^2\theta = 1} \Rightarrow \underline{1 - \cos^2\theta = \sin^2\theta}$

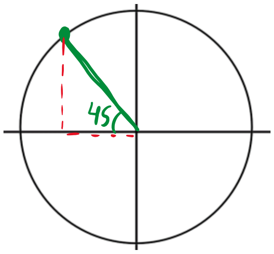
$\sin^2\theta = \sin^2\theta$ ↑ Pythagorean Identity

32. Find the coordinates (x, y) on the unit circle for the given radian measure. *Circle provided if needed.

$\theta = -\frac{5\pi}{4}$ $\left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}} \right)$

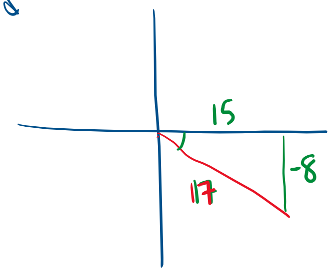
$\frac{\sqrt{2}}{2} \quad \frac{\sqrt{2}}{2}$

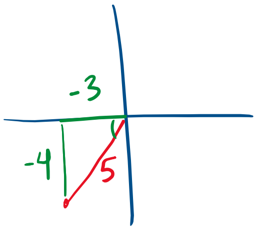




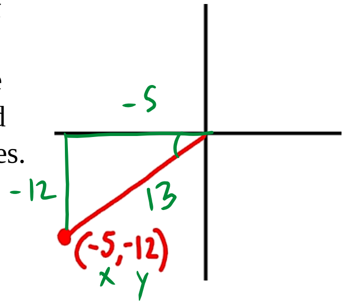
33. For each, find the indicated trig value in the specified quadrant. ***Write answers as fractions!***

Function	**Quadrant**	**Value Desired**
$\sin(\theta) = -8/17$	IV	$\tan(\theta) = -8/15$
$\cot(\theta) = 3/4$ $\tan(\theta) = 4/3$	III	$\sin(\theta) = -4/5$





34. Given the coordinates of the point are on the terminal side of an angle in standard position, find all six trig function values.



$\sin\theta = \frac{-12}{13}$ $\quad \cos\theta = \frac{-5}{13}$ $\quad \tan\theta = \frac{12}{5}$

$\csc\theta = -\frac{13}{12}$ $\quad \sec\theta = -\frac{13}{5}$ $\quad \cot\theta = \frac{5}{12}$