

Chapter 7 Notes

7.1 – Applying Exponent Product Properties

Lead-In: In 2003, the US Department of Agriculture (USDA) collected data on 10^3 bee colonies. If each colony contains 10^4 bees, how many bees did the USDA include in their study?

Exponents are used to communicate repeated _____

a^2	a^3	$a^2 \cdot a^3$
$6n^3 \cdot 2n^7$	$(3pt^3)(p^3t^4)$	<u>Product of Powers Rule</u>

Power of Powers Property

$(x^2)^3$	$(y^3)^5$	<u>Power of Power Property</u>
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Example: Simplify $[(2^3)^2]^4$.

A 2^{24}

B 2^{12}

C 2^{10}

D 2^9

Power of a Product Property

$(6x)^4$	$(4m^2n)^3$	<u>Power of Product Property</u>
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Challenging Ones!!!

$((x^2y)^6)^5$	$(3xy^4)^2[(-2y)^2]^3$
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7.2 – Applying Exponent Properties Involving Quotients, Negative Exponents and Zero Exponents

Lead-In: To measure the brightness (luminosity) of a star, we measure how much power it puts out, in watts. Our Sun produces 10^{26} W, and Canopus has a luminosity of 10^{30} W. How many times more power does Canopus produce?

$\frac{a^5}{a^3}$	<i>Example:</i>	<u>Quotient of Powers Rule</u>
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All Exponent Properties

$a^m \cdot a^n = a^{m+n}$	$(a^m)^n = a^{mn}$	$(ab)^m = a^m b^m$	$\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$	$\frac{a^m}{a^n} = a^{m-n}$
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Examples

$\frac{5^3 \cdot 5^5}{5^2}$	$\left(\frac{2a^4b^3}{ab^2}\right)^2$	$\frac{2s^3t^3}{st^2} \cdot \frac{(3st)^3}{s^2t}$
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a. Tabular Fill in the table from left to right

Power	3^4	3^3	3^2	3^1	3^0	3^{-1}	3^{-2}	3^{-3}	3^{-4}
Value									

Negative and Zero Exponents

$$a^0 =$$

$$a^{-n} =$$

$$\frac{1}{a^{-n}} =$$

What about $\frac{1}{4^{-1}}$?

Example



<p><u>Method 1</u></p> $\frac{-8x^2y^8z^{-5}}{12x^4y^{-7}z^7} = \left(\frac{-8}{12}\right)\left(\frac{x^2}{x^4}\right)\left(\frac{y^8}{y^{-7}}\right)\left(\frac{z^{-5}}{z^7}\right)$ $= \left(\frac{-2}{3}\right)(x^{2-4})(y^{8-(-7)})(z^{-5-7})$	<p><u>Method 2</u></p> $\frac{-8x^2y^8z^{-5}}{12x^4y^{-7}z^7}$	$\frac{(5pr^{-2})^{-2}}{(3p^{-1}r)^3}$	$\left(-\frac{3xy^4z^2}{x^3yz^4}\right)^0$
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7.3 – Rational Exponents

$(\sqrt{5})^2 =$ _____ , so $\sqrt{5}$ is equal to 5 to what power?

What if this was the cube root of 5?

What if this was the nth root of 5?


 KeyConcept $b^{\frac{1}{2}}$	
Words	For any nonnegative real number b , $b^{\frac{1}{2}} = \sqrt{b}$.
Examples	$16^{\frac{1}{2}} = \sqrt{16}$ or 4 $38^{\frac{1}{2}} = \sqrt{38}$
 KeyConcept $b^{\frac{1}{n}}$	
Words	For any positive real number b and any integer $n > 1$, $b^{\frac{1}{n}} = \sqrt[n]{b}$.
Example	$8^{\frac{1}{3}} = \sqrt[3]{8} = \sqrt[3]{2 \cdot 2 \cdot 2}$ or 2

Write each in radical form or with an exponent. $25^{\frac{1}{2}}$ $2\sqrt{x}$	Simplify and state the answer. $\sqrt[3]{27}$ $\sqrt[5]{32}$
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If we can do fractions like 1/2, 1/3, and 1/4, how do we interpret exponents with other fractions?

$64^{\frac{2}{3}}$

$36^{\frac{3}{2}}$

 KeyConcept $b^{\frac{m}{n}}$	
Words	For any positive real number b and any integers m and $n > 1$, $b^{\frac{m}{n}} = (\sqrt[n]{b})^m$ or $\sqrt[n]{b^m}$.
Example	$8^{\frac{2}{3}} = (\sqrt[3]{8})^2 = 2^2$ or 4

Example: Putting all the pieces together.

$(8^2)^{2/3}$	$(81^{1/4})^{-2}$	Solve for x. $9^x = 27$
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7.5 Day 1 – Write and Graph Exponential Growth Functions

Lead – In : You have \$10 in a bank account and are given each of the following options:

- a) \$50 will be added to the account each week b) The account will double each week

Which is the better deal after 4 weeks? After 6 weeks?

Weeks	0	1	2	3	4	5	6
\$							

Weeks	0	1	2	3	4	5	6
\$							

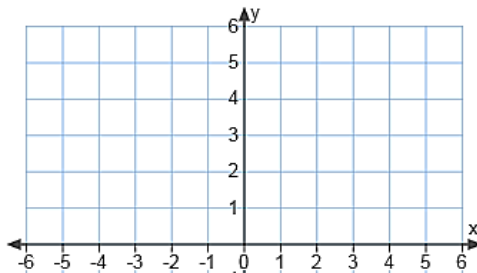
Example: Write an equation for the function.

x	-2	-1	0	1	2
y	-4	-1	2	5	8

x	-2	-1	0	1	2
y	2	4	8	16	32

Example: Graph the function $y = 2^x$ and identify the domain and range. Note: $y = 2^x = 1(2^x)$

x	y
-2	
-1	
0	
1	
2	

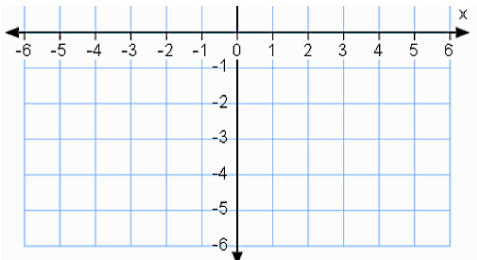


Domain:

Range:

Example: Graph the function $y = -4(1.5)^x$ and identify the domain and range.

x	y
-2	
-1	
0	
1	
2	



Domain:

Range:

Example: The function $C = 179(1.029)^t$ models the amount of soda, in billions of liters, consumed in the world, where t is the years after 2000.

- What does the y-intercept represent in this context?
- What are realistic domain and ranges?
- Use the equation to estimate the amount of soda consumed throughout the world this year.

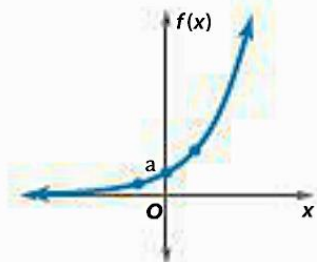
Exponential Growth Functions

Equation: $f(x) = ab^x$, $a > 0$, $b > 1$

Domain: _____ **Range:** _____

y-intercept: _____ **x-intercept:** _____

End behavior: as x increases, $f(x)$ _____
as x decreases, $f(x)$ _____



7.5 Day 2 – Write and Graph Exponential Decay Functions

Lead-In: Take a piece of yarn 1yd long and cut it in half. Continue this process.

Stage	Number of Pieces	Length of Each Piece
0	1	1
1		
2		
3		
4		

- Write a function for the number of pieces at stage x .
- Write a function for the length of each piece at stage x .

Example: Write the equation for the exponential function.

x	-2	-1	0	1	2
y	32	16	8	4	2

Exponential Growth Functions:

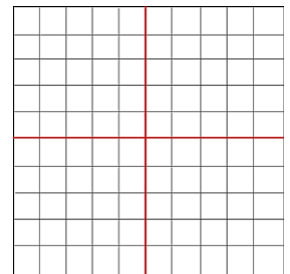
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Exponential Decay Functions:

Ex:

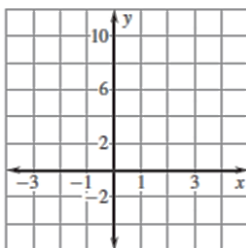
Example:

Graph $f(x) = 2\left(\frac{1}{2}\right)^x$ and $g(x) = 2(2)^x$ on the graph to the right and compare.



Example: Graph and state the domain and range of each.

$$y = 2 \cdot \left(\frac{1}{5}\right)^x$$

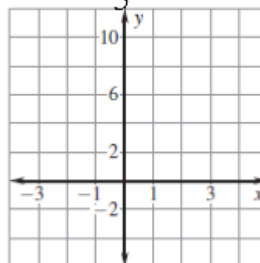


x			0		
y					

Domain:

Range:

$$y = 2\left(\frac{1}{5}\right)^x - 3$$



x			0		
y					

Domain:

Range:

Example: Determine whether the set of data shown below displays exponential behavior. If so, write an exponential equation. If not, explain why not.

x	-2	-1	0	1	2
y	-4/3	-4	-12	-36	-108

x	-2	-1	0	1	2
y	-18	-12	-6	0	6

x	-4	-2	0	2	4
y	1/16	1/4	1	4	16

7.6 – Growth and Decay Equations

Modeling Population Growth

The population of Helena in the year 2000 was 23,000. The U.S. Census Bureau found that the population then increased, on average, by 2% each year. What was the population in 2005?

Year	0	1	2	3	4	5
Population						

Example: An investor places \$250,000 in an account that earns 4% interest each year. How much will it be worth in 5 years?

KEY CONCEPT *For Your Notebook*

Exponential Growth Model

a is the **initial amount**. r is the **growth rate**.

$1 + r$ is the **growth factor**. t is the **time period**.

$$y = a(1 + r)^t$$

Decay Example: A car is purchased for \$18,996. The car depreciates at a rate of 15% per year. After 6 years, the owner is offered \$7500 for it. Is this a fair deal?

KEY CONCEPT *For Your Notebook*

Exponential Decay Model

a is the **initial amount**. r is the **decay rate**.

$1 - r$ is the **decay factor**. t is the **time period**.

$$y = a(1 - r)^t$$

Exponential Situations Example: Write a formula for each situation.

A rare coin is purchased from a dealer at a value of \$300. The value of the coin increases 7.5% each year.	There is originally 5.4 grams of a radioactive element that decays at a rate of 4.69% per year.	In 1850, there were 20 million bison in the United States, but they began to decline by 2% each year.
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Compound Interest:

KeyConcept Equation for Compound Interest

A is the current amount. n is the number of times the interest is compounded each year, and t is time in years.

P is the principal or initial amount. r is the annual interest rate expressed as a decimal, $r > 0$.

$$A = P\left(1 + \frac{r}{n}\right)^{nt}$$

Example: Re-answer the savings account example from above if the interest is compounded:

Monthly:

Daily:

Example: Match the graph to its equation.

$$y = 2(0.2)^x$$

$$y = 2(0.8)^x$$

$$y = (1.5)^x$$

