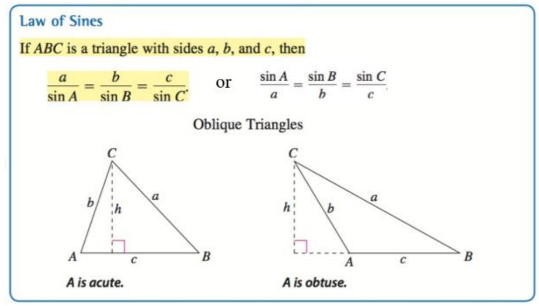
Honors Precalculus Name: ___________________________________

Chapter 6 Notes

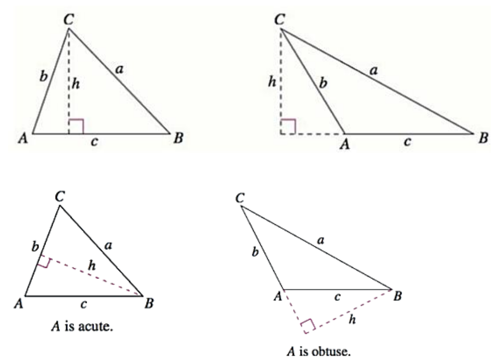
6.1 Day 1 Proving the Law of Sines and Basic Examples

In order to solve for missing sides/angles of a non-right triangle (oblique), you have to use either the Law of Sines or Law of Cosines. The Law of Sines is used when you have 1) Two angles and _____ side (AAS or ASA) or 2) Two sides and an angle ___________________ one of them (SSA).

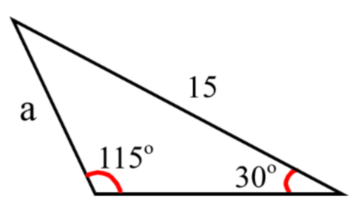


Proof for the Law of Sines

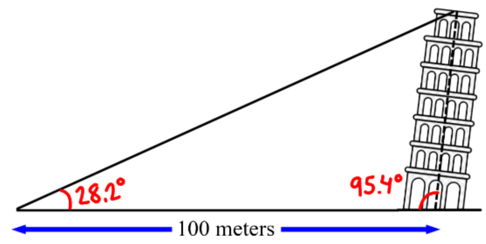
Let *h* be the altitude of each triangle.



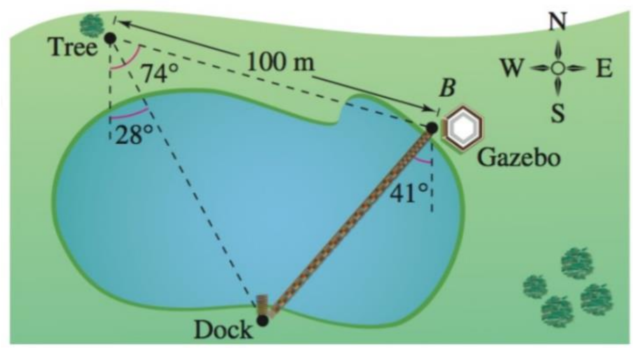
Example: Solve the triangle using the Law of Sines.



Example: The Leaning Tower of Pisa leans 5.4° past vertical because it was built on unstable soil. If its angle of elevation 28.2° when measured from 100 meters, what is its height?



Example: A bridge is to be built across a small lake from the gazebo to the dock. How long will it be?

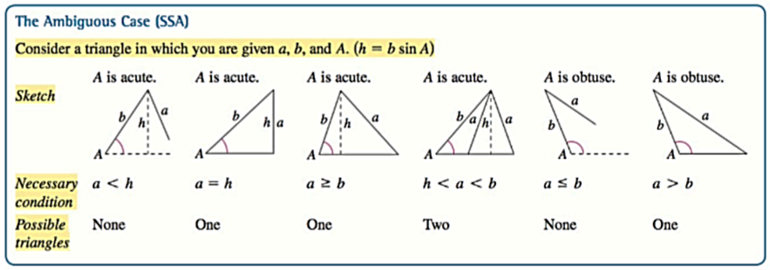


6.1 Day 2 Ambiguous Cases and Areas of Oblique Triangles using Law of Sines

Note: in the Law of Sines the height can be found by using $h =$ ____________________

When using the Law of Sines with SSA, there are 6 possibilities.

For each below, state the number of possible triangles and if there is at least one, make a sketch and solve for the other angles and side of each triangle.



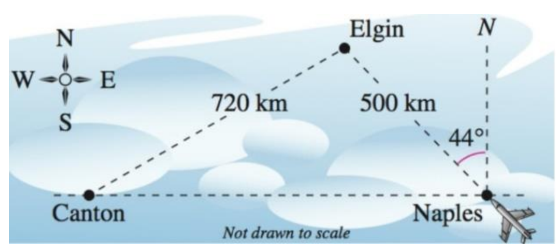
The Ambiguous Case (SSA)

Consider a triangle in which you are given a, b, and A. ($h = b \sin A$)

Sketch	A is acute.	A is acute.	A is acute.	A is acute.	A is obtuse.	A is obtuse.
Necessary condition	$a < h$	$a = h$	$a \geq b$	$h < a < b$	$a \leq b$	$a > b$
Possible triangles	None	One	One	Two	None	One

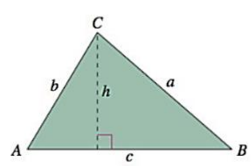
a = 24, b = 15, and A = 26°	a = 15, b = 25, and A = 85°
a = 12, b = 31, and A = 20.5°	

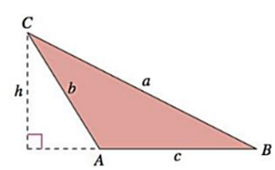
Ex: Find the bearing of the flight from Elgin to Canton.



Rewriting the Area of a Triangle

$A = \frac{1}{2}(base)(height)$





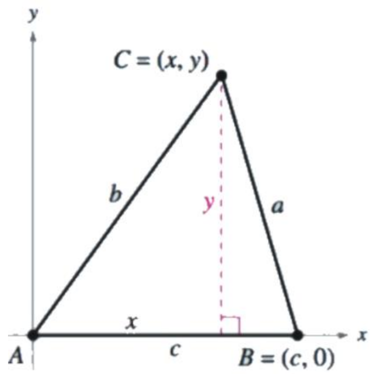
$A =$

6.2 Day 1 Proving Law of Cosines and Basic Examples Use Law of Cosines for SSS and SAS situations

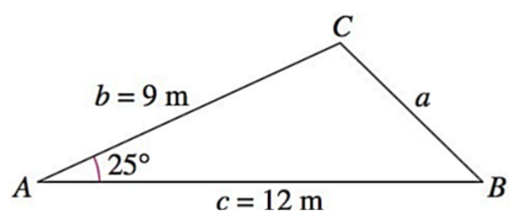
Proof for the Law of Cosines

Law of Cosines

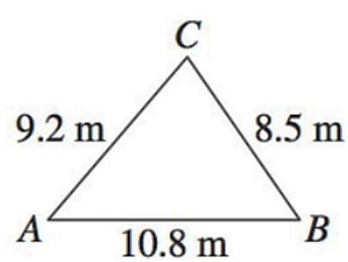
Standard Form	*Alternative Form*
$a^2 = b^2 + c^2 - 2bc\cos A$	$\cos A = \dfrac{b^2 + c^2 - a^2}{2bc}$
$b^2 = a^2 + c^2 - 2ac\cos B$	$\cos B = \dfrac{a^2 + c^2 - b^2}{2ac}$
$c^2 = a^2 + b^2 - 2ab\cos C$	$\cos C = \dfrac{a^2 + b^2 - c^2}{2ab}$



Ex: Solve the triangle with SAS.



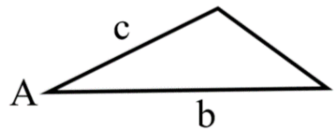
Ex: Solve the triangle when given SSS.



6.2 Day 2 Proving Heron's Formula and Applications of Law of Cosines

Proof of Heron's Formula

$A = \frac{1}{2} bc \cdot sinA$



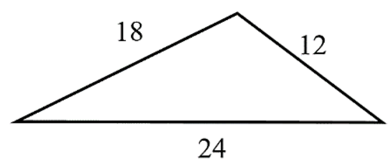
Heron's Area Formula

Given any triangle with sides of lengths a, b, and c, the area of the triangle is

$$\text{Area} = \sqrt{s(s-a)(s-b)(s-c)}$$

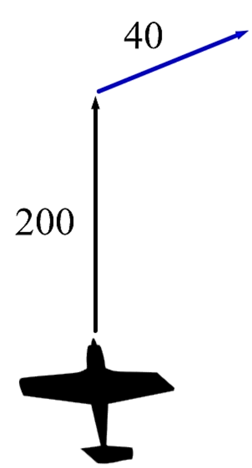
where $s = \dfrac{a+b+c}{2}$.

Ex: Use Heron's formula to find the area of the following triangle.



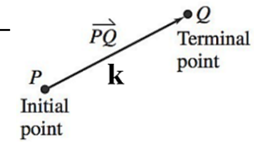
Ex: Application of Law of Cosines

A small plane travelling due north at 200 mph is pushed by a 40 mph wind with a bearing of E 25° N, as shown. Find the speed the plane is now actually moving with the wind assisting.



<u>**6.3 Day 1 Vector Operations**</u>

<u>**Vector**</u>: a quantity having ________________________ and _____________________



Component Form of a Vector

The component form of the vector with initial point $P(p_1, p_2)$ and terminal point $Q(q_1, q_2)$ is given by

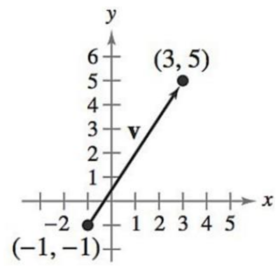
$$\overrightarrow{PQ} = \langle q_1 - p_1, q_2 - p_2 \rangle = \langle v_1, v_2 \rangle = \mathbf{v}.$$

The **magnitude** (or **length**) of **v** is given by

$$\|\mathbf{v}\| = \sqrt{(q_1 - p_1)^2 + (q_2 - p_2)^2} = \sqrt{v_1^2 + v_2^2}.$$

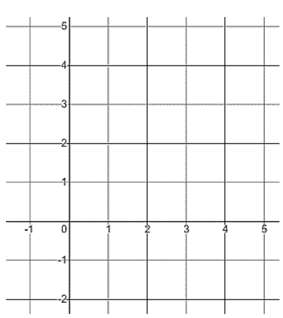
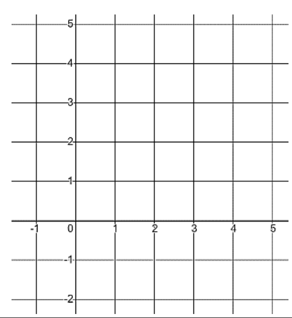
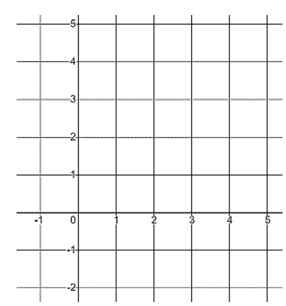
If $\|\mathbf{v}\| = 1$, then **v** is a **unit vector.** Moreover, $\|\mathbf{v}\| = 0$ if and only if **v** is the zero vector **0**.

<u>**Ex**</u>: Find the component form and magnitude of the vector **v**



<u>**Vector Addition**</u>	<u>**Vector Scalar Multiplication**</u>
$\boldsymbol{u} + \boldsymbol{v} =$	$k\boldsymbol{u} =$

Let **w** = <3, 2> and **v** = <-1, 4>. Find each algebraically and make a sketch.

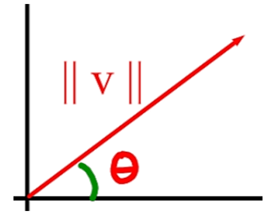
2**w**	**w** - **v**	**v** + 2**w**
		

<u>**Unit vectors**</u>: the vectors $< 1,0 >$ and $< 0,1 >$ can be denoted by unit vectors **i**$=< 1,0 >$ and **j**$=< 0,1 >$

Let $\vec{u}$ be the vector with initial point (-2, 6) and terminal point (-8,3). Write $\vec{u}$ using unit vectors $\vec{\imath}$ and $\vec{\jmath}$.	Let **u** = 4**i** + 2**j** and **v** = -2**i** - 5**j**. Find 3**u** – 6**v**.

6.3 Day 2 Direction Angles, Finding Component Form, and Speed Direction

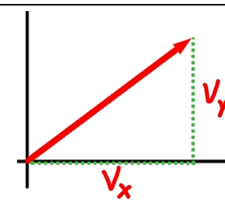
For a vector **v** with magnitude $\|v\|$ and direction θ, you can find its components as follows:



Ex: For the given magnitude and direction, write each in component form.

$\|v\| = 10$ and $\theta = 30°$	$\|v\| = 10$ and $\theta = 120°$

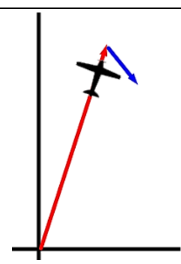
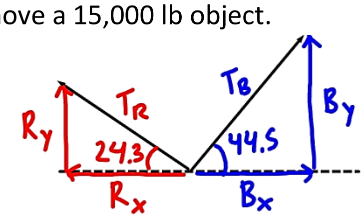
If you are given component form and need to work backwards to get the magnitude and direction for a vector $\boldsymbol{v} = < v_x, v_y >$.



$\|v\| =$

$\tan(\theta) =$

Ex: If **u** = -4**i** + 7**j**, find the magnitude and direction of **u**.

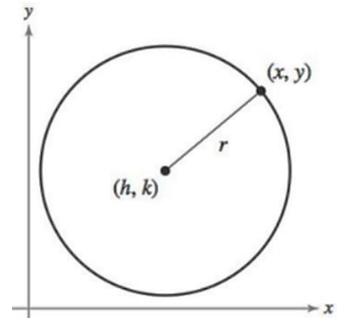
A plane traveling at a speed of 500 mph with a bearing of N 30° E encounters a wind of 75 mph coming from S 45° E. What is the plane's new speed and direction? 	Two cranes are helping move a 15,000 lb object. Determine the tension in each. 

9.1 Equation for a Circle Derive and Apply

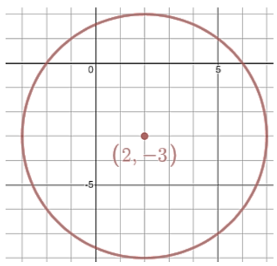
Circle: set of all points (x,y) in the plane that are ________________ from a fixed center, (h,k).

Deriving the Equation for a Circle

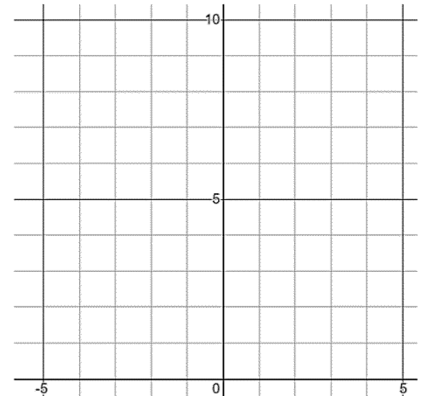
$distance\ from\ (h,k) to\ (x,y) =$



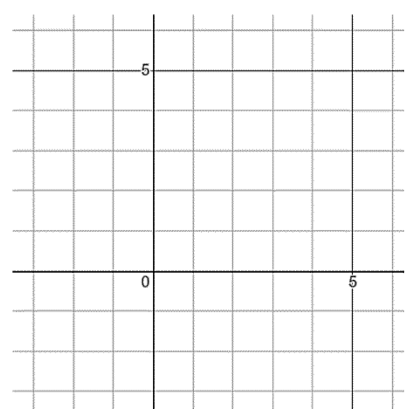
Write the equation for the circle shown.



Sketch the graph for $(x+1)^2 + (y-5)^2 = 16$



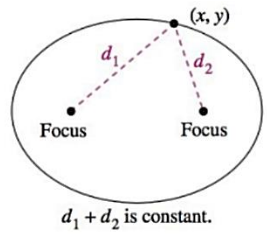
Write the equation of the circle in standard form and then sketch its graph.
$x^2 - 2x + y^2 - 4y - 4 = 0$

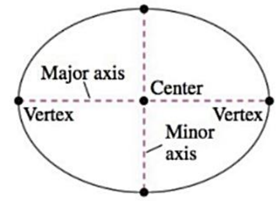


Find the x and y-intercept(s) of the graph of the circle (round to nearest hundredth).
$(x+2)^2 + (y-1)^2 = 10$

9.2 Equation for an Ellipse Derive, Apply, and Eccentricity

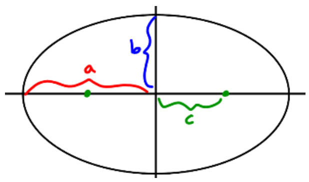
Ellipse: set of all points in a plane, the _________ of whose distances from two distinct fixed points (foci) is constant





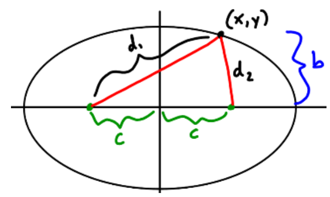
Deriving the Equation for an Ellipse

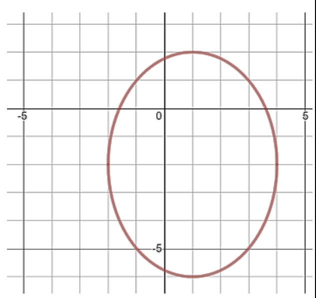
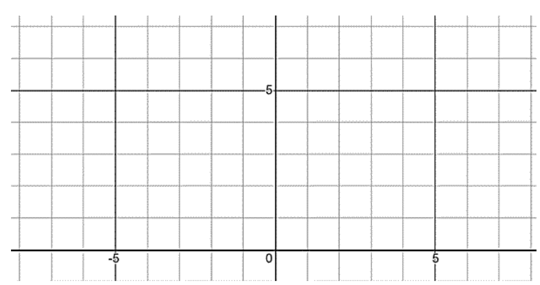
For simplicity, consider the ellipse shown on the left.



Note, when the point on the ellipse is directly at the top, we get $b^2 + c^2 = a^2$ since the string's entire length is 2a.

Now, if the point is anywhere else on the ellipse we get:

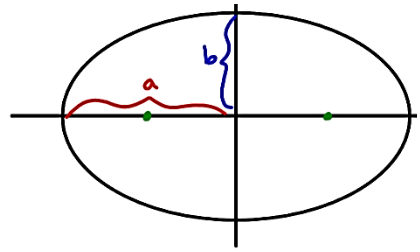


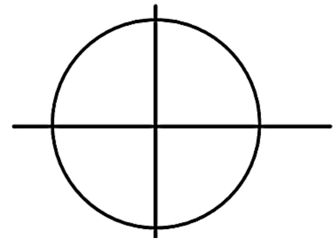
<table>
<tr>
<td>Write the equation for the ellipse shown.
</td>
<td>Sketch the graph for $\frac{x^2}{49} + \frac{(y-3)^2}{9} = 1$
</td>
</tr>
<tr>
<td colspan="2">Write the equation for the ellipse that has vertices (3, 1) and (3,9) and a minor axis length of 6.</td>
</tr>
</table>

Relating Back to a Circle

We know that a circle is just a special case of an ellipse, so how can you get the equation for a circle from the equation for an ellipse?

$$\frac{(x-h)^2}{a^2}+\frac{(y-k)^2}{b^2}=1$$





Recall how we showed one could construct an ellipse using two thumbtacks, a string of fixed length, paper, and a pencil.

1) Where would you place the foci to make an ellipse that is very elliptical (long and narrow)?

2) Where would you place the foci to make an ellipse that is nearly circular?

<u>Eccentricity</u>: denoted *e*, it is a measure computed by $e = \frac{c}{a}$, which determines how "elliptical" a figure is.

What are the max and minimum values for *e*?

The orbit of the moon has an eccentricity of

$e \approx 0.0554$ Eccentricity of the moon

and the eccentricities of the eight planetary orbits are as follows.

Planet	Eccentricity, *e*
Mercury	0.2056
Venus	0.0068
Earth	0.0167
Mars	0.0934
Jupiter	0.0484
Saturn	0.0539
Uranus	0.0473
Neptune	0.0086

9.5 Graphing Polar Coordinates and Coordinate Conversion

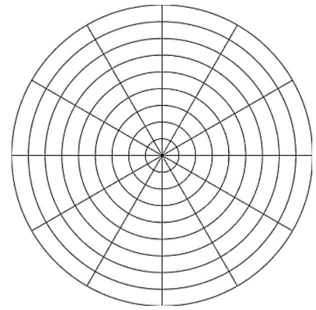
Polar Coordinates: plot coordinates according to their coordinates of (radius, _____________)

Plot and label each point given in polar coordinates.

$A:\left(3, \frac{\pi}{6}\right)$ $B:\left(3, \frac{\pi}{4}\right)$

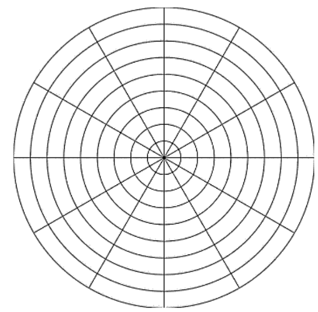
$C:\left(5, -\frac{5\pi}{3}\right)$ $D:\left(-7, \frac{\pi}{6}\right)$

$E:\left(3, -\frac{7\pi}{4}\right)$

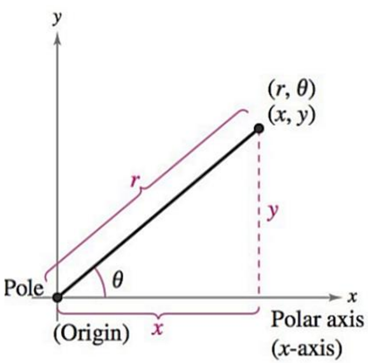


Plot the point provided and find three additional polar representations (between -2π and 2π) for the same point.

$(4, \frac{7\pi}{6})$



Converting Between Polar and Rectangular



Coordinate Conversion

The polar coordinates (r, θ) are related to the rectangular coordinates (x, y), as follows.

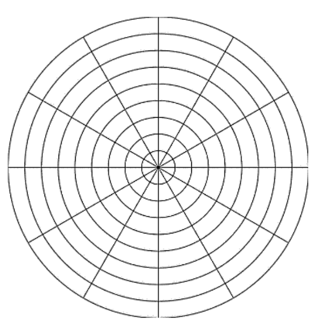
Polar-to-Rectangular | *Rectangular-to-Polar*

$x =$ | $\tan \theta =$

$y =$ | $r^2 =$

Plot each of the points and then convert their coordinates to rectangular.

$A:\left(4, \frac{5\pi}{6}\right)$ $B:\left(6, -\frac{\pi}{4}\right)$



Plot each of the points and then convert their coordinates to polar.

$A: (\sqrt{3}, -1)$ $B: (3, -2)$