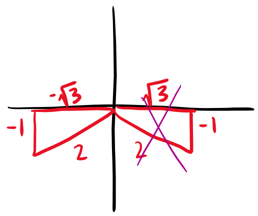
Honors Precalculus

Name:______________________________ Per: _____

Chapter 5 PRACTICE TEST *Use an additional sheet, if necessary, to show your work*

1. Use the fact that $\sin x = -1/2$ and $\tan x > 0$ to get the other 5 trig values.



$\sin(x) = -\frac{1}{2}$ $\cos(x) = \frac{-\sqrt{3}}{2}$ $\tan(x) = \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}$

$\csc(x) = -2$ $\sec(x) = -\frac{2}{\sqrt{3}} = \frac{-2\sqrt{3}}{3}$ $\cot(x) = \sqrt{3}$

2. Verify #1 by factoring and #2 by using sines & cosines.

a) $\sin^2 x \sec x - \sec x = -\cos x$

$\sin^2 x + \cos^2 x = 1$

$\sin^2 - 1 = -\cos^2 x$

$\sec x(\sin^2 x - 1) = -\cos x$

$\sec x(-\cos^2 x) = -\cos x$

$\frac{1}{\cancel{\cos x}}(-\cancel{\cos^2 x}) = -\cos x \Rightarrow \boxed{-\cos x = -\cos x}$ ✓

b) $\csc t \sec t - \sin t \sec t = \cot t$

$\frac{1}{\sin t}\frac{1}{\cos t} - \sin t \frac{1}{\cos t} = \cot t$

$\frac{1}{\sin t}\frac{1}{\cos t} - \frac{\sin^2 t}{\sin t}\frac{1}{\cos t} = \cot t$

$\frac{1-\sin^2 t}{\sin t \cos t} = \cot t$

$\frac{\cancel{\cos^2 t}}{\sin t \cancel{\cos t}} = \cot t$

$\frac{\cos t}{\sin t} = \cot t$

$\boxed{\cot t = \cot t}$ ✓

3. Use conjugates to show $\frac{\sin^2\theta}{1+\cos\theta} = 1-\cos\theta$

$\frac{\sin^2\theta(1-\cos\theta)}{(1+\cos\theta)(1-\cos\theta)} = 1-\cos\theta$

$\frac{\sin^2\theta(1-\cos\theta)}{1-\cos^2\theta} = 1-\cos\theta$

$\frac{\cancel{\sin^2\theta}(1-\cos\theta)}{\cancel{\sin^2\theta}} = 1-\cos\theta$

$\boxed{1-\cos\theta = 1-\cos\theta}$ ✓

4. Verify $\frac{\cot x \tan x}{\sin x} = \csc x$

$\frac{\frac{1}{\cancel{\tan x}} \cdot \frac{\cancel{\tan x}}{1}}{\sin x} = \csc x$

$\frac{1}{\sin x} = \csc x$

$\boxed{\csc x = \csc x}$ ✓

5. Verify by using an identity for $\csc^2 x$ first.

$$\frac{\csc^2 x - \cot^2 x}{\tan^2 x \csc^2 x} = \cos^2 x$$

Use $1+\cot^2 x = \csc^2 x$

$\frac{1+\cot^2 x - \cot^2 x}{\tan^2 x \csc^2 x} = \cos^2 x$

$\frac{1}{\tan^2 x \csc^2 x} = \cos^2 x$

$\frac{1}{\frac{\cancel{\sin^2 x}}{\cos^2 x} \cdot \frac{1}{\cancel{\sin^2 x}}} = \cos^2 x$

$\frac{1}{\frac{1}{\cos^2 x}} = \cos^2 x$

$\frac{1}{\sec^2 x} = \cos^2 x$

$\boxed{\cos^2 x = \cos^2 x}$

6. Verify $\frac{1}{\sec\theta - 1} + \frac{1}{\sec\theta + 1} = 2\cot\theta\csc\theta$

$\frac{1}{(\sec\theta - 1)}\frac{(\sec\theta+1)}{(\sec\theta+1)} + \frac{1}{(\sec\theta+1)}\frac{(\sec\theta-1)}{(\sec\theta-1)} = 2\cot\theta\csc\theta$

$1+\tan^2\theta = \sec^2\theta$

$\tan^2\theta = \sec^2\theta - 1$

$\frac{\sec\theta \cancel{+1} + \sec\theta \cancel{-1}}{\sec^2\theta - 1} = 2\cot\theta\csc\theta$

$\frac{2\sec\theta}{\tan^2\theta} = 2\cot\theta\csc\theta$

$2\sec\theta \div \tan^2\theta = 2\cot\theta\csc\theta$

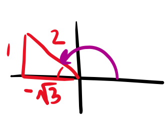
$2\frac{1}{\cancel{\cos\theta}} \cdot \frac{\cos^{\cancel{2}}\theta}{\sin^2\theta} = 2\cot\theta\csc\theta$

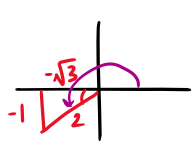
$2\frac{\cos\theta}{\sin\theta}\frac{1}{\sin\theta} = 2\cot\theta\csc\theta$

$\boxed{2\cot\theta\csc\theta = 2\cot\theta\csc\theta}$ ✓

7. Solve $2\cos x + \sqrt{3} = 0$ on the interval $[0, 2\pi)$.

$\cos x = \frac{-\sqrt{3}}{2}$



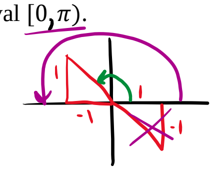


$x = \boxed{\frac{5\pi}{6}}$ $x = \boxed{\frac{7\pi}{6}}$

8. Solve $-\cot x - 1 = 0$ on the interval $[0, \pi)$.

$\cot x = -1$

$\frac{\cos x}{\sin x} = -1$

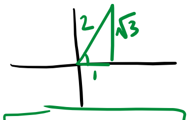


$\boxed{x = \frac{3\pi}{4}}$

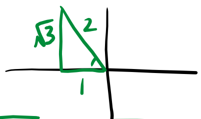
8. Solve $\sqrt{3}\csc x - 2 = 0$ for all solutions.

$$\csc x = \frac{2}{\sqrt{3}}$$

$$\sin x = \frac{\sqrt{3}}{2}$$



$$\boxed{x = \frac{\pi}{3} + 2\pi n} \quad n \in \mathbb{Z}$$



$$x = \boxed{\frac{2\pi}{3} + 2\pi n} \quad n \in \mathbb{Z}$$

9. Solve $\sin^2 x + \cos x + 1 = 0$ on the interval $[0, 2\pi)$.

$$\sin^2 x + \cos^2 x = 1$$
$$\sin^2 x = 1 - \cos^2 x$$

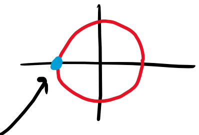
$$1 - \cos^2 x + \cos x + 1 = 0$$
$$-\cos^2 x + \cos x + 2 = 0$$
$$\cos^2 x - \cos x - 2 = 0$$
$$(\cos x - 2)(\cos x + 1) = 0$$

$$a^2 - a - 2 = 0$$
$$(a-2)(a+1) = 0$$

$\cos x - 2 = 0$ $\qquad$ $\cos x + 1 = 0$

$\cancel{\cos x = 2}$ impossible $\qquad$ $\cos x = -1$

$$\boxed{x = \pi}$$

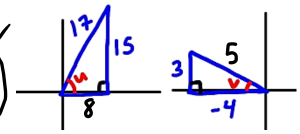


9. Use a sum and difference formula to find $\cos(15°)$ exactly (no decimals).

$$\cos(u - v) = \cos u \cos v + \sin u \sin v$$
$$\cos(45 - 30) = \cos(45)\cos(30) + \sin(45)\sin(30)$$
$$= \frac{\sqrt{2}}{2}\,\frac{\sqrt{3}}{2} + \frac{\sqrt{2}}{2}\,\frac{1}{2}$$
$$\cos(15) = \boxed{\frac{\sqrt{6} + \sqrt{2}}{4}}$$

10. Given the angles shown, find $\sin(u + v)$.

$$\sin(u+v) = \sin u \cos v + \cos u \sin v$$
$$= \left(\frac{15}{17}\right)\left(\frac{-4}{5}\right) + \left(\frac{8}{17}\right)\left(\frac{3}{5}\right)$$
$$= \frac{-60}{85} + \frac{24}{85}$$
$$= \boxed{\frac{-36}{85}}$$

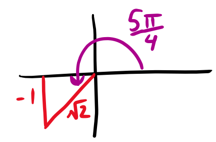


11. Use sum and difference formulas to find the two solutions on the interval $[0, 2\pi)$.

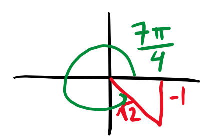
$$\sin\left(x + \frac{\pi}{4}\right) + \sin\left(x - \frac{\pi}{4}\right) = -1$$

$$\sin x \cos\frac{\pi}{4} + \cancel{\cos x \sin\frac{\pi}{4}} + \sin x \cos\frac{\pi}{4} - \cancel{\cos x \sin\frac{\pi}{4}} = -1$$
$$2 \sin x \cos\frac{\pi}{4} = -1$$
$$\cancel{2} \sin x \left(\frac{\sqrt{2}}{\cancel{2}}\right) = -1$$

$$\sqrt{2}\sin x = -1$$
$$\sin x = \frac{-1}{\sqrt{2}}$$



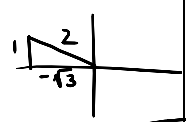
$$\boxed{x = \frac{5\pi}{4}} \qquad \boxed{x = \frac{7\pi}{4}}$$



12. Use a half-angle formula to find the exact value for $\cos(75°)$.

$u = 150°$

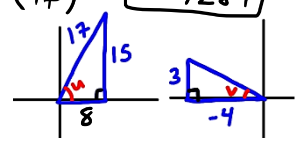
$$\cos\left(\frac{u}{2}\right) = \sqrt{\frac{1 + \cos u}{2}}$$
$$\cos\left(\frac{150}{2}\right) = \sqrt{\frac{1 + \cos(150)}{2}}$$



$$\cos(75) = \sqrt{\frac{1 - \frac{\sqrt{3}}{2}}{2}} = \sqrt{\frac{\frac{2}{2} - \frac{\sqrt{3}}{2}}{2}} = \sqrt{\frac{2 - \sqrt{3}}{4}} = \boxed{\frac{\sqrt{2 - \sqrt{3}}}{2}}$$

13. Use the figure from Problem 10 to find $\sin(2u)$ and $\cos(2u)$ with the double-angle formulas.

$$\sin 2u = 2\sin u \cos u = 2\left(\frac{15}{17}\right)\left(\frac{8}{17}\right) = \boxed{\frac{240}{289}}$$
$$\cos 2u = \cos^2 u - \sin^2 u = \left(\frac{8}{17}\right)^2 - \left(\frac{15}{17}\right)^2 = \boxed{-161/289}$$



14. The equation of a constant standing electric wave in physics is obtained by adding two waves traveling in opposite directions (y_1 *and* y_2). Given the equations for each individual wave, show $y_1 + y_2 = 2A\cos\frac{2\pi t}{T}\cos\frac{2\pi x}{\lambda}$.

$y_1 = A\cos\left(\frac{2\pi t}{T} - \frac{2\pi x}{\lambda}\right)$ and $y_2 = A\cos\left(\frac{2\pi t}{T} + \frac{2\pi x}{\lambda}\right)$

y_1 $\qquad$ + $\qquad$ y_2

$$A\left[\cos\frac{2\pi t}{T}\cos\frac{2\pi x}{\lambda} + \cancel{\sin\frac{2\pi t}{T}\sin\frac{2\pi x}{\lambda}}\right] + A\left[\cos\frac{2\pi t}{T}\cos\frac{2\pi x}{\lambda} - \cancel{\sin\frac{2\pi t}{T}\sin\frac{2\pi x}{\lambda}}\right]$$

$$\boxed{2A\cos\frac{2\pi t}{T}\cos\frac{2\pi x}{\lambda}}$$