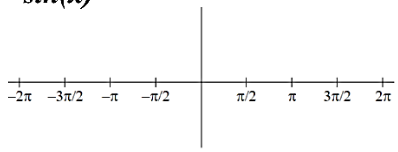
Honors Precalculus Name: ___________________________________

Chapter 4 Part 2 Notes

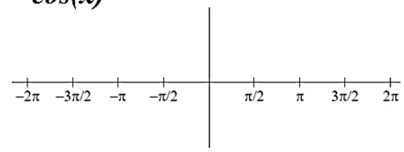
4.5 Day 1 – Graphs of Sine and Cosine

x	0	$\pi/2$	π	$3\pi/2$	2π
$sin(x)$					
$cos(x)$					

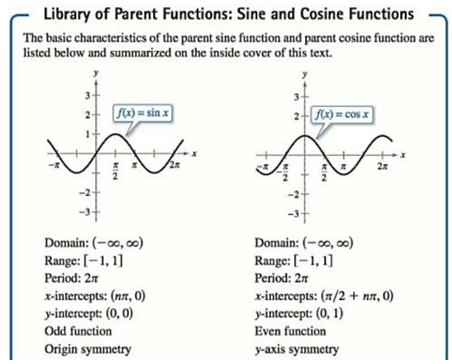
sin(x)



cos(x)



Amplitude: half the distance between the max and min values for a sine or cosine graph. It's always positive! For the parent function it is _____.



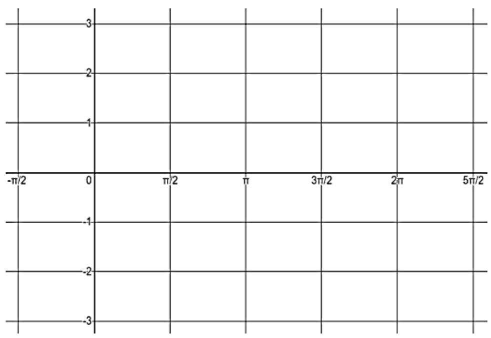
$y = a \cdot \sin x$
a acts as
$\|a\| > 1$ the basic sine curve is
$\|a\| < 1$ the basic sine curve is
$a < 0$ the parent functions is

f(x) = -0.5cos(x)

Amplitude =

g(x) = 3cos(x)

Amplitude =



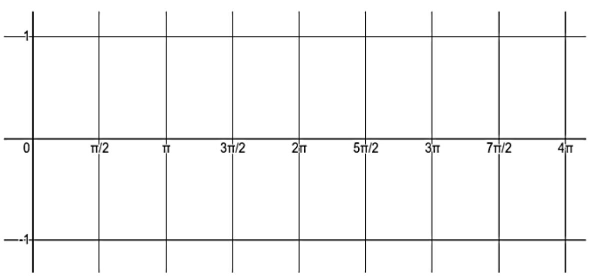
$y = \sin(bx)$
b acts as
Period is the time it takes to complete one fully cycle and go from peak to peak.
$Period =$
If $b < 0$ we apply the following identities $\sin(-x) =$ and $\cos(-x) =$

• *f(x) = sin(4x)*

Period =

• $g(x) = \sin\left(\frac{x}{2}\right)$

Period =



Note: once you have the period for each, you can find where the key points will occur by dividing the period by 4 since there are 4 key points.

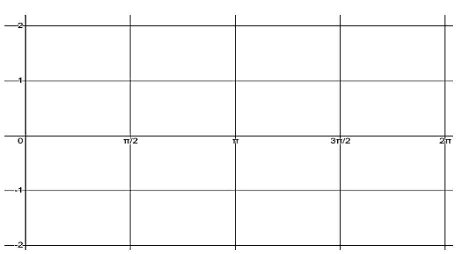
• Start Intercept (0, 0) (,) Intercept (,) (,) Intercept (,)

• Start Intercept (0, 0) (,) Intercept (,) (,) Intercept (,)

Sketch the graph for *g(x) = 2sin(3x)*

Amplitude =

Period =



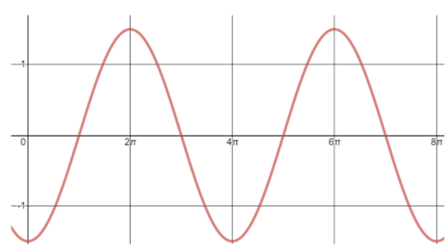
Check on your calculator

State what the equation is for the graph shown.

sine or cosine?

Amplitude =

Period =

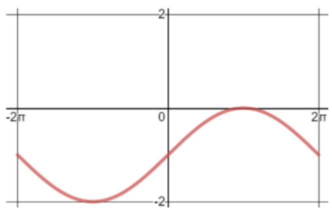


4.5 Day 2 – Horizontal and Vertical Translations and Sketching Graphs by Hand

$f(x) = \sin(x) + d$	$f(x) = \sin(x + c)$
d is a	*c is a*
$\sin(x) + 3$ would move the graph	$\sin(x + 3)$ would move the graph
$\sin(x) - 2$ would move the graph	$\sin(x - 2)$ would move the graph

*We'll come back to the question below @ the end of the lesson

Which equation matches the graph shown?



$f(x) = 2\sin(x) - 2$

$g(x) = \sin(0.5x) - 1$

$h(x) = \cos(0.5x) - 1$

$k(x) = 2\cos(x) - 2$

Summary

Transformation of Trigonometric Graphs

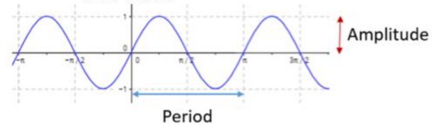
$y = A\sin\left[B(x - C)\right] + D$ ← you may have to factor B out to get this form

|A| is the amplitude

The period is $\frac{2\pi}{B}$

Phase (horizontal) shift is C

Vertical shift is D



The same applies for the Cosine Function.

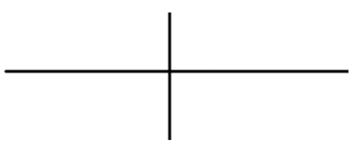
Identify the information and sketch the graph for each.

$$f(x) = sin\left(x - \frac{\pi}{2}\right) + 1$$

Horizontal Shift = Amplitude =

Period = Vertical Shift =

1) Sketch the parent function



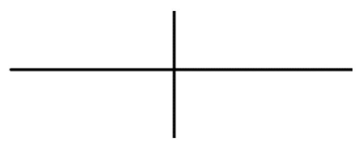
2) Perform the transformations one at a time, according to the order of operations.

$$f(x) = cos(x + \pi) - 1$$

Horizontal Shift = Amplitude =

Period = Vertical Shift =

1) Sketch the parent function



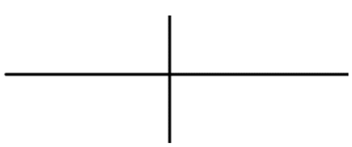
2) Perform the transformations one at a time, according to the order of operations.

$f(x) = sin(2x + \pi) =$

Horizontal Shift = Amplitude =

Period = Vertical Shift =

1) Sketch the parent function



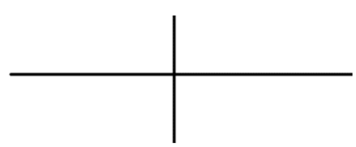
2) Perform the transformations.

$$f(x) = 0.5cos(x) + 2$$

Horizontal Shift = Amplitude =

Period = Vertical Shift =

1) Sketch the parent function



2) Perform the transformations.

4.5 Day 3 – Equations with All Four Transformations and Writing an Equation

Identify the information and sketch the graph for each.

$$f(x) = 2sin\left(x - \frac{\pi}{2}\right) - 1$$

Horizontal Shift = Amplitude =

Period = Vertical Shift =

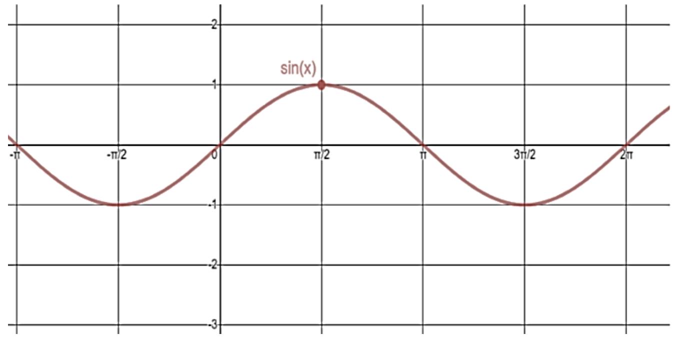
1) Use the parent function table for sin(x) to find the new x start coordinate by applying the horizontal shift.

x	0	$\pi/2$	π	$3\pi/2$	2π
$\sin x$	0	1	0	-1	0

x	$\pi/2$				
$f(x)$					

2) Since you have the period, you can find where the next 4 key points will occur by dividing the period by 4 and adding it to your start x - coordinate.

$$\frac{Period}{4} =$$



3) Apply the y-coordinate transformations, according to the order of operations, to get the new outputs.

Each y-coordinate from the parent function is multiplied by 2 and then subtract 1

$f(x) = -cos(2\pi x + 4\pi) + 1 =$

Horizontal Shift = Amplitude =

Period = Vertical Shift =

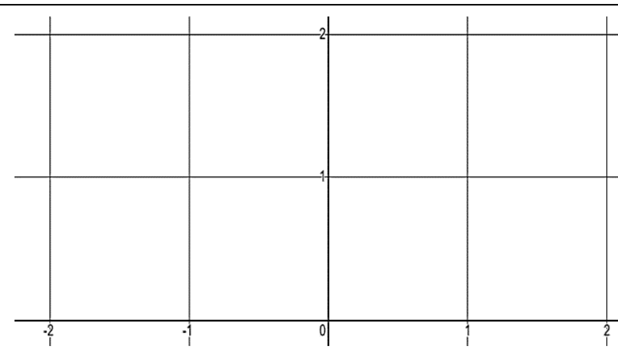
1) Use the parent function table for cos(x) to find the new x start coordinate by applying the horizontal shift.

x	0	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$	2π
$\cos x$	1	0	-1	0	1

x					
$f(x)$					

2) Since you have the period, you can find where the next 4 key points will occur by dividing the period by 4 and adding it to your start x - coordinate.

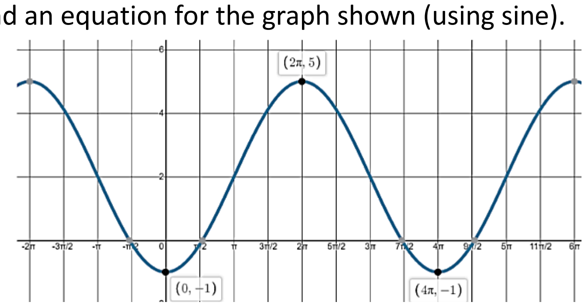
$$\frac{Period}{4} =$$



3) Apply the y-coordinate transformations, according to the order of operations, to get the new outputs.

Each y-coordinate from the parent function is multiplied by -1 and then added 1

Find an equation for the graph shown (using sine).



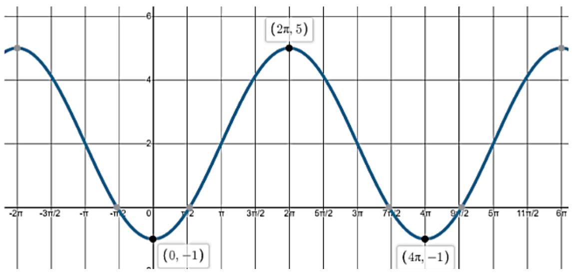
$$f(x) = a\sin\big(b(x + c)\big) + d$$

1) Pick 5 points that follow in the same order of the parent sine function (on the midline, max, on the midline, min, on the midline) determine the period and then "b".

2) Use the first of the five points to determine the horizontal shift ("c"). Use the new midline to determine the vertical shift ("d").

3) Use the max and min values to determine the amplitude since the amplitude is half the distance between.

Find an equation for the graph shown (using cosine).



$$f(x) = a\cos\big(b(x + c)\big) + d$$

1) Pick 5 points that follow in the same order of the parent sine function (max, on the midline, min, on the midline, max) determine the period and then "b".

2) Use the first of the five points to determine the horizontal shift ("c"). Use the new midline to determine the vertical shift ("d").

3) Use the max and min values to determine the amplitude since the amplitude is half the distance between.

Example: Application

(Source: U.S. Naval Observatory)

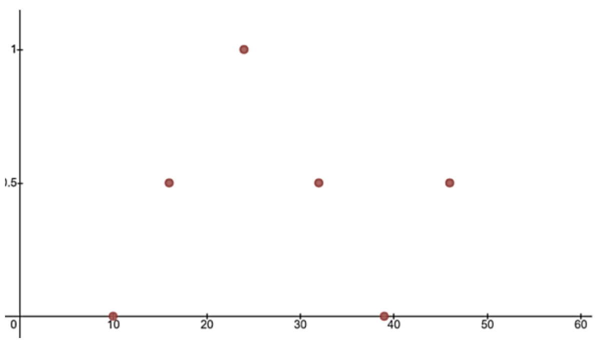
The table shows the percent (in decimal form) of the moon's face that is illuminated on day x of the year 2016, where x = 1 represents Jan. 1.

DATA

Spreadsheet at LarsonPrecalculus.com

Day, x	Percent, y
10	0.0
16	0.5
24	1.0
32	0.5
39	0.0
46	0.5

a) Create a scatter plot of the data.



b) Determine an equation that models the data set (use sine) and state what the period is and if that makes sense.

c) Use our equation to estimate the percent illumination of the moon for tonight.

Days between Jan. 1, 2016 and today =

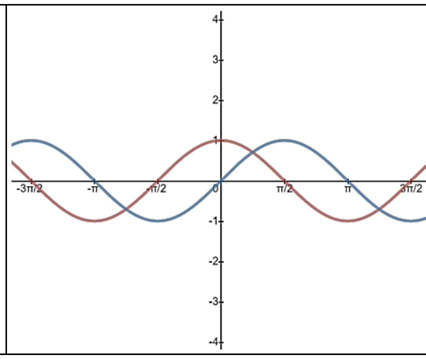
$p(\qquad) =$

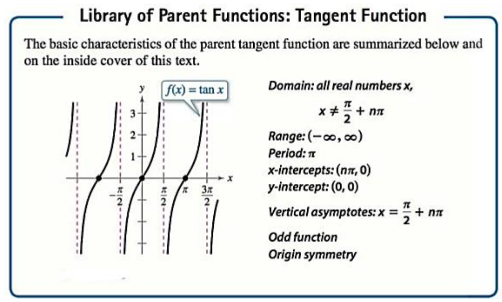
What the moon phase (percent illuminated) actually is today =

4.6 Day 1 – Graphs of Tangent and Cotangent

Graphing *tan(x)*: Use the fact that tan(x) = sin(x)/cos(x) to reason through what the graph for f(x) = tan(x) will look like.

$$f(x) = \tan x = \frac{\sin x}{\cos x}$$





Library of Parent Functions: Tangent Function

The basic characteristics of the parent tangent function are summarized below and on the inside cover of this text.

Domain: all real numbers x,

$x \neq \frac{\pi}{2} + n\pi$

Range: $(-\infty, \infty)$

Period: π

x-intercepts: $(n\pi, 0)$

y-intercept: $(0, 0)$

Vertical asymptotes: $x = \frac{\pi}{2} + n\pi$

Odd function

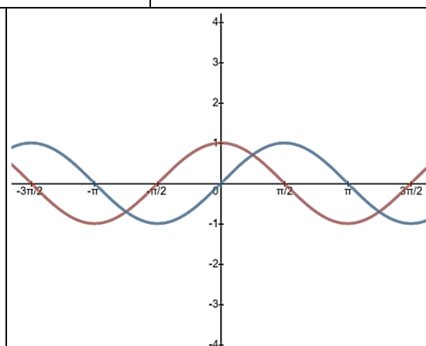
Origin symmetry

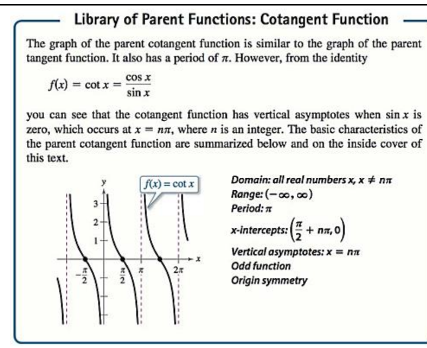
Describe in words how the following functions compare to the parent function f(x) = tan(x) and then sketch a graph.

$g(x) = tan\left(\frac{x}{2}\right)$	$g(x) = -tan(2x)$

Graphing *cot(x)*: Use the fact that cot(x) = cos(x)/sin(x) to reason through what the graph for f(x) = cot(x) will look like.

$$f(x) = \cot x = \frac{\cos x}{\sin x}$$





Library of Parent Functions: Cotangent Function

The graph of the parent cotangent function is similar to the graph of the parent tangent function. It also has a period of π. However, from the identity

$$f(x) = \cot x = \frac{\cos x}{\sin x}$$

you can see that the cotangent function has vertical asymptotes when $\sin x$ is zero, which occurs at $x = n\pi$, where n is an integer. The basic characteristics of the parent cotangent function are summarized below and on the inside cover of this text.

Domain: all real numbers x, $x \neq n\pi$

Range: $(-\infty, \infty)$

Period: π

x-intercepts: $\left(\frac{\pi}{2} + n\pi, 0\right)$

Vertical asymptotes: $x = n\pi$

Odd function

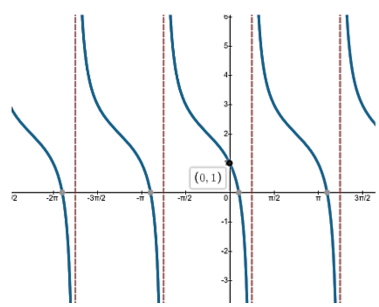
Origin symmetry

Describe in words how the following function compares to the parent function f(x) = cot(x) and then sketch a graph.

$$g(x) = cot\left(\frac{x}{3}\right)$$

Determine the values for c and d in the equation for the graph shown. $f(x) = \cot(x + c) + d$

1) to determine "c" note the shift of the vertical asymptotes.



2) to determine d, plug the y-intercept into the equation and solve for d.

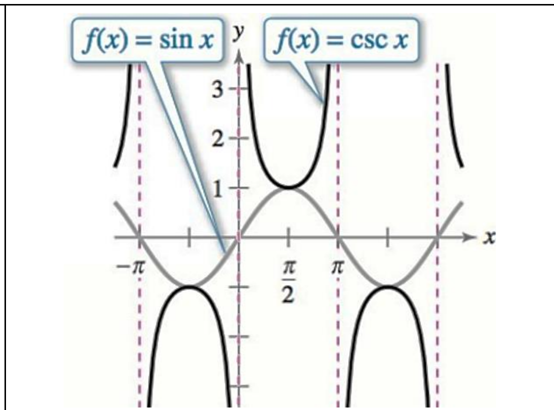
4.6 Day 2 – Graphs of Cosecant, Secant, and Damped Trig Graphs

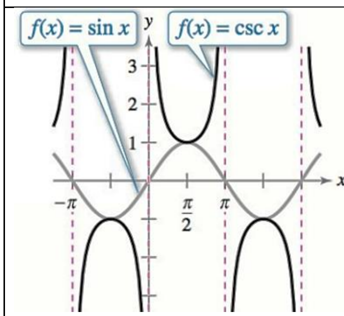
Graph of the Cosecant Function

Recall: $\csc x = \frac{1}{\sin x}$

The graph for csc(x) is shown in black (the graph of sin(x) is shown in gray). Explain why the graph of this makes sense.

- csc(x) will have vertical asymptotes where ___________ = 0
- Where sin(x) is a _________________ is where csc(x) will have its minimum and vice versa, since they are __________________ of one another
- Where sin(x) is _____________, csc(x) will be positive as well





Summary of $\csc x$

***Domain:** all real numbers x, $x \neq n\pi$*

***Range:** $(-\infty, -1] \cup [1, \infty)$*

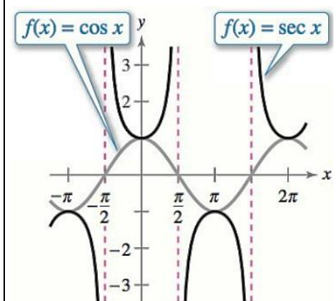
***Period:** 2π*

No intercepts

***Vertical asymptotes:** $x = n\pi$*

Odd function

Origin symmetry



Summary of $\sec x$

***Domain:** all real numbers x, $x \neq \frac{\pi}{2} + n\pi$*

***Range:** $(-\infty, -1] \cup [1, \infty)$*

***Period:** 2π*

***y-intercept:** (0, 1)*

***Vertical asymptotes:** $x = \frac{\pi}{2} + n\pi$*

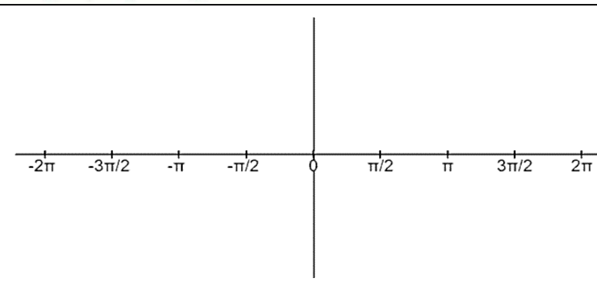
Even function

y-axis symmetry

Sketch the graph for $f(x) = 2csc\left(x - \frac{\pi}{2}\right)$

1) First, sketch the graph for $y = 2\sin\left(x - \frac{\pi}{2}\right)$

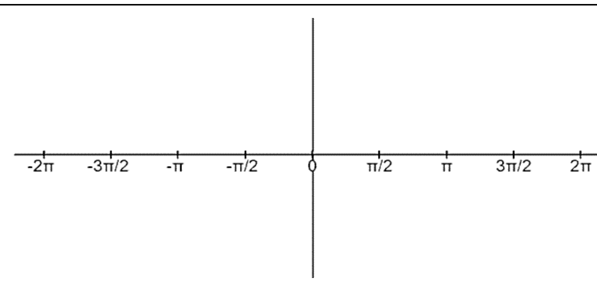
2) Use the graph for sine to sketch in the vertical asymptotes, mins, and maxes of the cosecant function and then finish the graph.



Sketch the graph for $f(x) = -sec(2x)$

1) First, sketch the graph for $y = -\cos(2x)$

2) Use the graph for cosine to sketch in the vertical asymptotes, mins, and maxes of the secant function and then finish the graph.



Application: Pendulums

Which graph, sine or cosine, would model the pendulum's distance from its center location (letting distance to the right be positive and distance to the left be negative) over time?

What if we had air resistance (friction)? The graph would then look like:

This is called a _____________________ graph and can be modeled by the following equation.

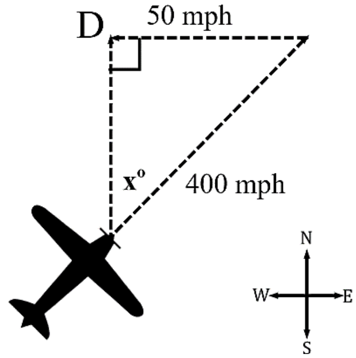
4.8 – Solve Right Triangles and Trig Applications

$\sin(Angle) = opp/hyp$ $\sin^{-1}(opp/hyp) = Angle$ You use the inverse trig functions when you have the

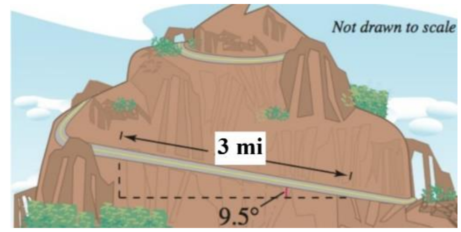
$\cos(Angle) = adj/hyp$ $\cos^{-1}(adj/hyp) = Angle$ ratio, but want to know the angle!

$\tan(Angle) = opp/hyp$ $\tan^{-1}(opp/adj) = Angle$ $\sin(45°) = 1/\sqrt{2}$ so $\sin^{-1}(1/\sqrt{2}) =$ __________

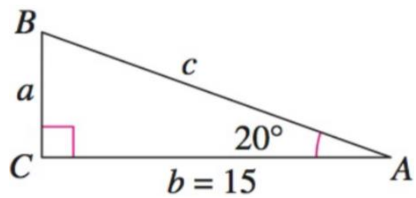
What direction should the plane fly (called its bearing) so that the wind from the east will push it directly towards destination D?



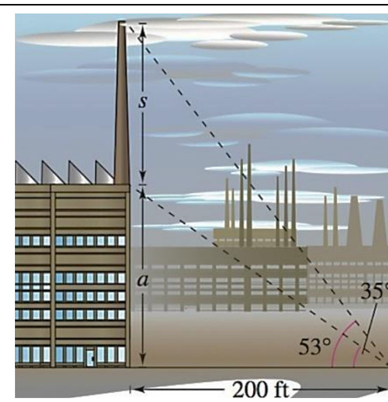
A sign on the roadway of a mountain indicates a 9.5° grade over the next 3 miles. What is the actual change in elevation?



Solve the triangle (find all sides and angles)

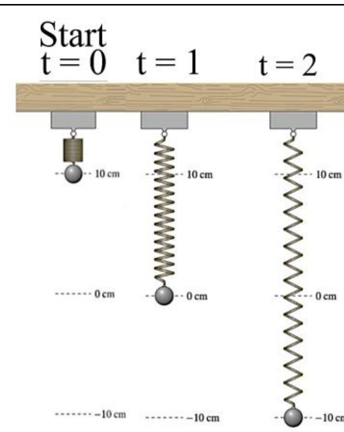


Set up two trig equations and solve for the height of the smokestack, s.



Consider the motion of the following mass on a spring that starts compressed and has a period of 4 seconds. Write the equation for its displacement from center using the simple harmonic motion equation.

> **Definition of Simple Harmonic Motion**
>
> A point that moves on a coordinate line is in **simple harmonic motion** when its distance d from the origin at time t is given by either
>
> $d = a \sin \omega t$ or $d = a \cos \omega t$
>
> where a and ω are real numbers such that $\omega > 0$. The motion has amplitude $|a|$, period $2\pi/\omega$, and frequency $\omega/(2\pi)$.



4.7 Day 1 – Evaluating Inverse Trig Functions and Their Graphs

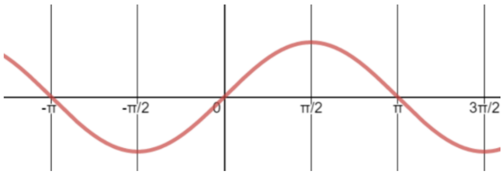
Inverses: functions *f(t)* and $f^{-1}(t)$ are inverses if the input of *f* is the output of f^{-1} and the output of *f* is the input of f^{-1}.

So, if $\sin\left(\frac{\pi}{6}\right) = \frac{1}{2}$ then we know .

Note: you will see the inverse sine function, $\sin^{-1}(x)$, also called $\arcsin(x)$.

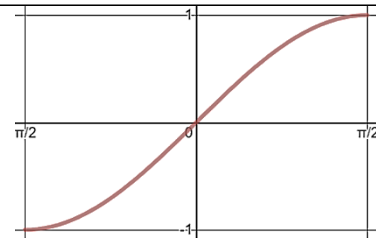
$y = \arcsin x$ or $y = \sin^{-1}(x)$

If you consider the graph of sin(x) along its entire domain, will it have an inverse? Why or why not? Can we fix this?



Ex: Use the table for f(x) = sin(x) along the restricted domain to make the table and graph for g(x) = arcsin(x).

x	$-\pi/2$	$-\pi/4$	$-\pi/6$	0	$\pi/6$	$\pi/4$	$\pi/2$
$\sin x$	-1	$-\frac{\sqrt{2}}{2}$	$-1/2$	0	$1/2$	$\frac{\sqrt{2}}{2}$	1



x							
$\sin^{-1} x$							

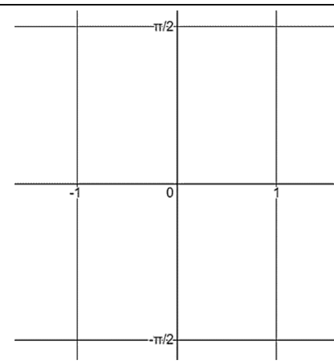
Domain: $[-1, 1]$

Range: $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

Intercept: $(0, 0)$

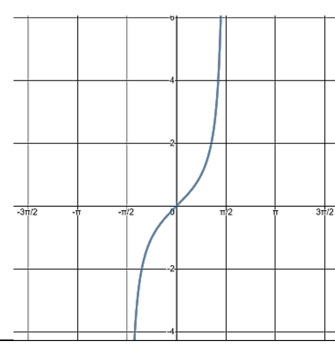
Odd function

Origin symmetry



Ex: Use the table for f(x) = tan(x) along the restricted domain to make the table and graph for g(x) = arctan(x).

x	$-\pi/2$	$-\pi/4$	$-\pi/6$	0	$\pi/6$	$\pi/4$	$\pi/2$
$\tan x$	$-\infty$	-1	$-\frac{\sqrt{3}}{3}$	0	$\frac{\sqrt{3}}{3}$	1	∞



x							
$\tan^{-1} x$							

Domain: $(-\infty, \infty)$

Range: $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

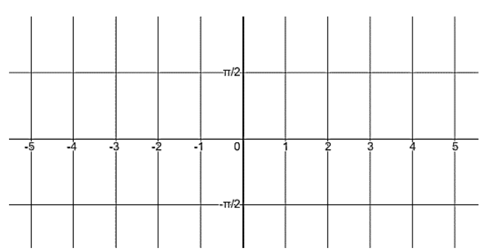
Intercept: $(0, 0)$

Intercept: $(0, 0)$

Horizontal asymptotes: $y = \pm\frac{\pi}{2}$

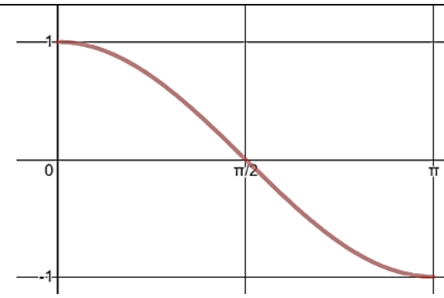
Odd function

Origin symmetry



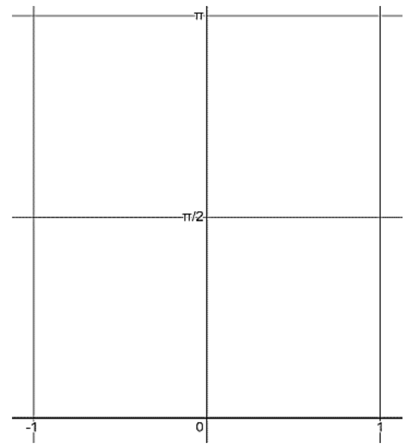
<u>Ex</u>: Use the table for f(x) = cos(x) along the restricted domain to make the table and graph for g(x) = arccos(x).

x	0	$\pi/6$	$\pi/3$	$\pi/2$	$2\pi/3$	$5\pi/6$	π
$\cos x$	1	$\frac{\sqrt{3}}{2}$	$1/2$	0	$-1/2$	$-\frac{\sqrt{3}}{2}$	-1



x							
$\cos^{-1} x$							

Domain: $[-1, 1]$
Range: $[0, \pi]$
y-intercept: $\left(0, \frac{\pi}{2}\right)$

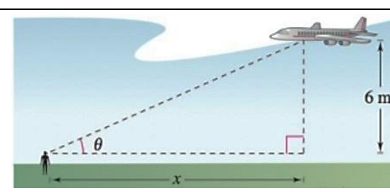


<u>Examples:</u> Determine the following. Sketch a picture if necessary and use the ranges of the inverse trig functions to determine the correct answer. Then check with a calculator.

$\cos^{-1}\left(-\frac{\sqrt{2}}{2}\right)$	$\arctan(1)$
$\arcsin\left(-\frac{1}{2}\right)$	$\cos^{-1}(\sqrt{3})$

4.7 Day 2 – Using Inverse Properties with Compositions

Compositions of Functions
Recall from Section 1.6 that for all x in the domains of f and f^{-1}, inverse functions have the properties
$f(f^{-1}(x)) = x$ and $f^{-1}(f(x)) = x$.
Inverse Properties
If $-1 \le x \le 1$ and $-\pi/2 \le y \le \pi/2$, then
$\sin(\arcsin x) = x$ and $\arcsin(\sin y) = y$.
If $-1 \le x \le 1$ and $0 \le y \le \pi$, then
$\cos(\arccos x) = x$ and $\arccos(\cos y) = y$.
If x is a real number and $-\pi/2 < y < \pi/2$, then
$\tan(\arctan x) = x$ and $\arctan(\tan y) = y$.
	Note Keep in mind that these properties do not apply for arbitrary values of x and y. For instance,
	$\arcsin\left(\sin \frac{3\pi}{2}\right) = \arcsin(-1) = -\frac{\pi}{2} \neq \frac{3\pi}{2}$.
	In other words, the property $\arcsin(\sin y) = y$ is not valid for values of y outside the interval $[-\pi/2, \pi/2]$.
	If this happens, you need to convert the given angle to an equivalent one (coterminal angle) in the range first.

$\sin(\sin^{-1}(1/2)) =$	$\arctan\left(tan\left(\frac{\pi}{4}\right)\right) =$
$\sin^{-1}\left(sin\left(\frac{5\pi}{3}\right)\right) =$	$\cos(arcos(\pi)) =$
$\sin(\tan^{-1}(4/3)) =$ If you stated what this is asking in English, it would say, "What is the sine ratio if the tangent ratio is 4/3?" Note, the range for arctan is [-π/2 , π/2], so we know this is in the first quadrant.	$\sec(\sin^{-1}(-5/13)) =$ If you stated what this is asking in English, it would say, "What is the secant ratio if the sine ratio is -5/13?" Note, the range for arcsin is [-π/2 , π/2], so we know this is in the first quadrant.
$\tan(\cos^{-1}(x/5)) =$ If you stated what this is asking in English, it would say, "What is the tangent ratio if the cosine ratio is x/5?"	a) Write θ as a function of *x*.  b) Find θ when *x = 10 miles* and *x = 3 miles*.