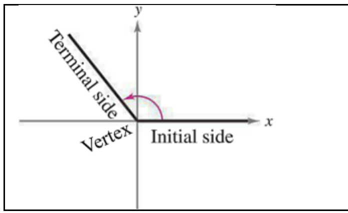
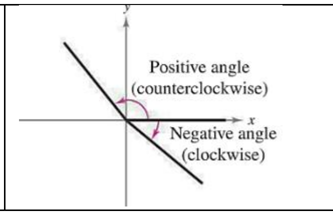
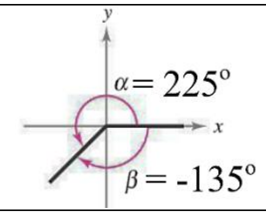
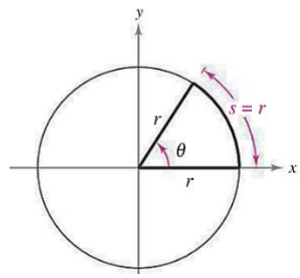
Honors Precalculus Name: ___________________________________

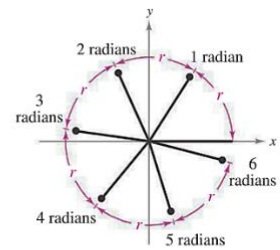
Chapter 4 Notes

4.1 Day 1 – Radians and Converting Between Them

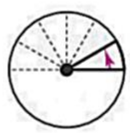
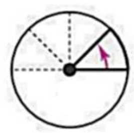
		**Coterminal**: angles that have the same initial and ____________________________ side. 

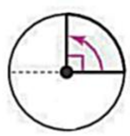
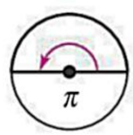
Radian: one radian is the measure of a central angle, ?, that creates an arc, s, equal in length to _______ ___________________ of the circle, r. Mathematically, the formula to determine the radian measurement is $\theta = s/r$.

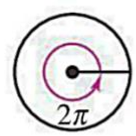




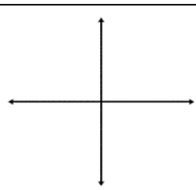
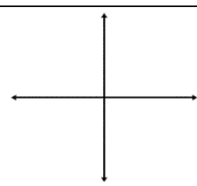
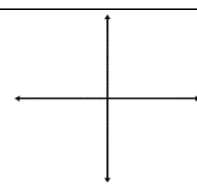
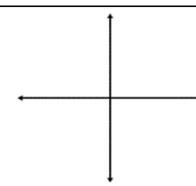
Example: Fill in the measurements below in radians.

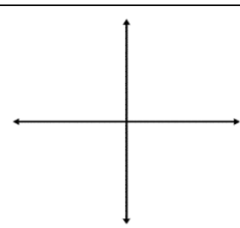
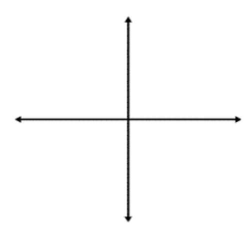





Example: Graph each of the angles below.

$\theta = \frac{-3\pi}{4}$ 	$\theta = \frac{5\pi}{3}$ 	$\theta = \frac{7\pi}{6}$ 	$\theta = \frac{5\pi}{4}$ 

Example: For each a) graph, b) state what quadrant they are in, and c) find two coterminal angles.

$\theta = \frac{3\pi}{4}$ 	$\theta = -\frac{2\pi}{3}$ 

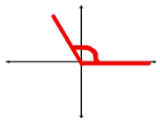
Continue to next side

Converting Between Radians and Degrees

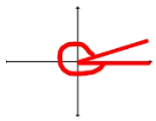
$\pi\ radians =$ __________° $\quad 2\pi\ radians =$ __________° $\quad 1\ radian \approx$ __________°

Example: Convert between radians and degrees.

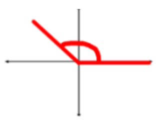
a) $\frac{2\pi}{3}$



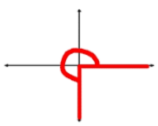
b) $-\frac{11\pi}{6}$



c) $135°$



d) $-270°$



Finding the Complement and Supplement (note: complementary angles sum to 90° and supplementary to 180°)

Ex: For each angle given, find the complement and supplement of each angle, if possible. Note, by definition, complements and supplements must be positive.

$\theta = \frac{\pi}{4}$	$\theta = \frac{4\pi}{5}$

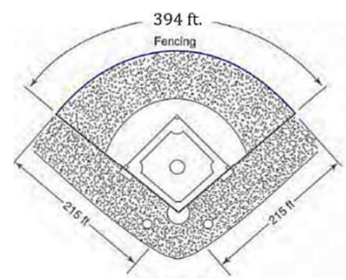
4.1 Day 2 – Linear and Angular Speed

Recall, we determine the radian measure through the formula $\theta = s/r$, so we can determine the arc length, s, by solving for s and obtaining the formula $s =$. Note, θ is measured in radians, NOT degrees!

Ex: As Earth rotates, how far do you travel through space in 8 hrs just based on this? Earth's radius ~ 4000 miles.

Ex: Find the distance between the cities (assume the Earth is a sphere of radius 4000 miles and the cities are on the same longitude line). Helena: 46° 35' 45'' N   and   Salt Lake City: 40° 45' 31'' N

Ex: The boundaries for the fence (which extend partway into foul territory) should be set at what angle (in degrees)?



Ex: The ISS makes an orbit every 90 minutes. If it orbits at a height of 254 miles and Earth's radius is 4000 miles, find its linear speed in miles/hour.

Linear and Angular Speed

Consider a particle moving at a constant speed along a circular arc of radius r. If s is the length of the arc traveled in time t, then the **linear speed** of the particle is

$$\text{Linear speed} = \frac{\text{arc length}}{\text{time}} = \frac{s}{t}.$$

Moreover, if θ is the angle (in radian measure) corresponding to the arc length s, then the **angular speed** of the particle is

$$\text{Angular speed} = \frac{\text{central angle}}{\text{time}} = \frac{\theta}{t}.$$

Linear speed measures how fast the particle moves, and angular speed measures how fast the angle changes.

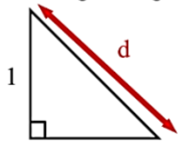
Ex: If the Ferris Wheel shown in the lesson has a radius of 30 feet, find

a) Angular speed of a passenger (rads/min)	b) Linear speed of a passenger (feet/min)

4.3 Day 1 – Right Triangle Trig

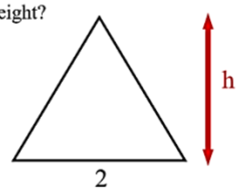
Lead In:

1) For the right isosceles triangle shown, what is the diagonal length?



a) 2 b) $\sqrt{2}$ c) 1 d) $\sqrt{3}$

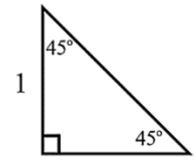
2) For the equilateral triangle shown, what is its height?



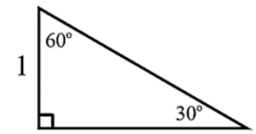
a) 2 b) $2\sqrt{2}$ c) 3 d) $\sqrt{3}$

Special Right Triangles

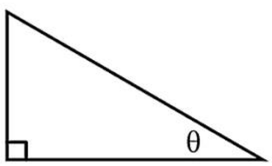
45-45-90



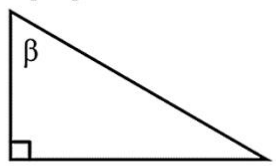
30-60-90



Consider the right triangles shown below from the perspective of the angle. Label the three sides as hypotenuse, opposite, and adjacent.



Re-label the 3 sides from β's perspective.



Six Trig Ratios

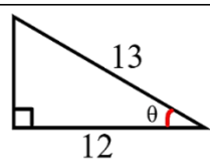
Let θ be an acute angle of a right triangle. Then:

$\sin(\theta) =$ $\csc(\theta) =$

$\cos(\theta) =$ $\sec(\theta) =$

$\tan(\theta) =$ $\cot(\theta) =$

Use the triangle shown to write the value of the six trig ratios/functions.



$\sin(\theta) =$ $\csc(\theta) =$

$\cos(\theta) =$ $\sec(\theta) =$

$\tan(\theta) =$ $\cot(\theta) =$

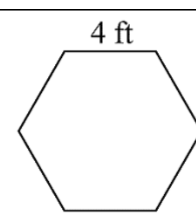
Sines, Cosines, and Tangents of Special Angles

$\sin 30° = \sin\frac{\pi}{6} =$	$\cos 30° = \cos\frac{\pi}{6} =$	$\tan 30° = \tan\frac{\pi}{6} =$
$\sin 45° = \sin\frac{\pi}{4} =$	$\cos 45° = \cos\frac{\pi}{4} =$	$\tan 45° = \tan\frac{\pi}{4} =$
$\sin 60° = \sin\frac{\pi}{3} =$	$\cos 60° = \cos\frac{\pi}{3} =$	$\tan 60° = \tan\frac{\pi}{3} =$

Examples

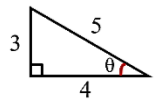
Evaluate sin(43° 40' 12") using a calculator.

Determine the distance across for the hot tub shown.



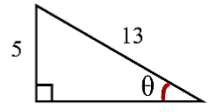
4.3 Day 2 – Trig Identities and Applications

For each of the triangles, find the following. Then state what you notice and wonder.



$\sin^2(\theta) =$

$+ \ \cos^2(\theta) =$



$\sin^2(\theta) =$

$+ \ \cos^2(\theta) =$

Fundamental Trigonometric Identities

Reciprocal Identities

* $\sin\theta = \frac{1}{\csc\theta}$ $\quad \cos\theta = \frac{1}{\sec\theta}$ $\quad \tan\theta = \frac{1}{\cot\theta}$

$\csc\theta = \frac{1}{\sin\theta}$ $\quad \sec\theta = \frac{1}{\cos\theta}$ $\quad \cot\theta = \frac{1}{\tan\theta}$

Quotient Identities

* $\tan\theta = \frac{\sin\theta}{\cos\theta}$ $\quad \cot\theta = \frac{\cos\theta}{\sin\theta}$

Pythagorean Identities

* $\sin^2\theta + \cos^2\theta = 1$

$1 + \tan^2\theta = \sec^2\theta$

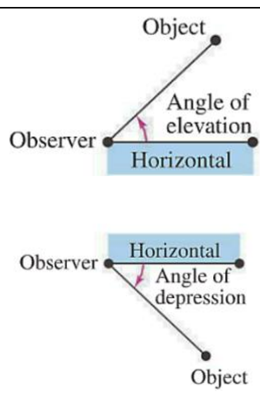
$1 + \cot^2\theta = \csc^2\theta$

We will prove the ones noted in red, but the rest are similar.

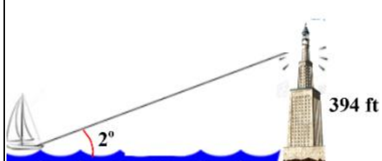
Proofs of Three Trig Identities

$\sin(\theta) = \frac{1}{\csc(\theta)}$	$\tan(\theta) = \frac{\sin(\theta)}{cos(\theta)}$	$\sin^2(\theta) + \cos^2(\theta) = 1$

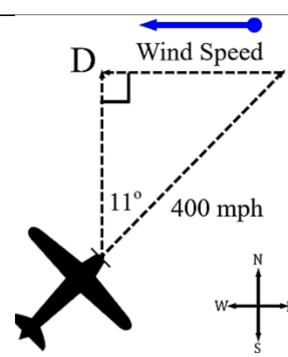
Let θ be an acute angle such that $\sin(\theta) = 0.6$. Find the values of $\cos(\theta)$ and $\tan(\theta)$ using trig identities.	Prove using trig identities. $\cot(\theta)\sin(\theta) = \cos(\theta)$	Prove using trig identities. $(1 + \cos\theta)(1 - \cos\theta) = \sin^2\theta$



Find the distance from the ship to the lighthouse.

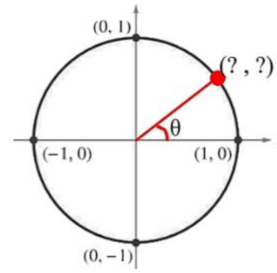


What was the speed of the wind?



4.2 Day 1 – Trig Functions on the Unit Circle

Lead-In: Consider a point on the circle shown that makes an angle θ with the vertex and x-axis. What are its x and y coordinates?



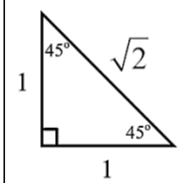
Definitions of Trigonometric Functions

Let θ be the angle made by a point, with coordinates (x, y), on the unit circle, the vertex of the unit circle, and the x-axis. Then, the following are true.

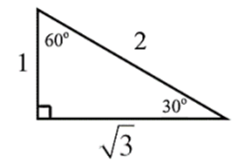
$\sin(\theta) =$ $\quad$ $\cos(\theta) =$ $\quad$ $\tan(\theta) =$

$\csc(\theta) =$ $\quad$ $\sec(\theta) =$ $\quad$ $\cot(\theta) =$

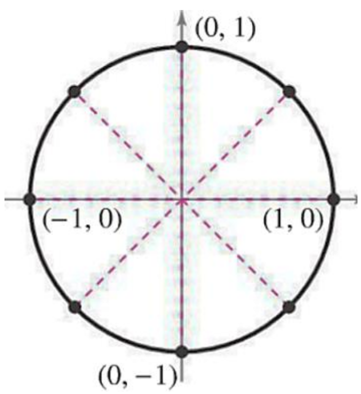
45-45-90



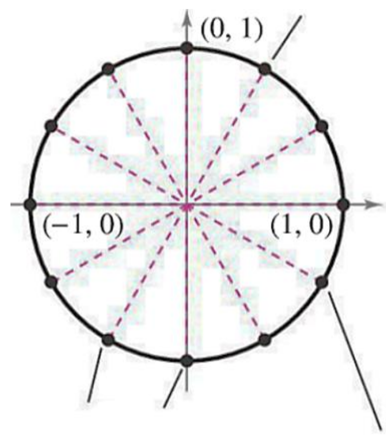
30-60-90



Unit Circle Trig Values for $\pi/4$ (45°) Intervals

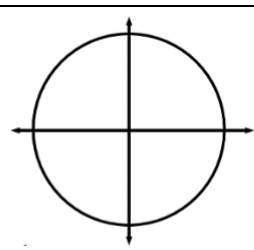


Unit Circle Trig Values for $\pi/6$ (30°) Intervals



Ex: Evaluate each of the six trigonometric functions at the given angle.

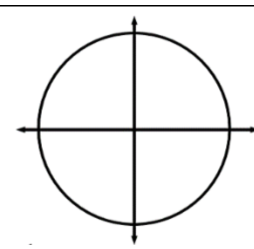
$\theta = \frac{5\pi}{4}$



$\sin(\theta) =$ $\quad$ $\cos(\theta) =$ $\quad$ $\tan(\theta) =$

$\csc(\theta) =$ $\quad$ $\sec(\theta) =$ $\quad$ $\cot(\theta) =$

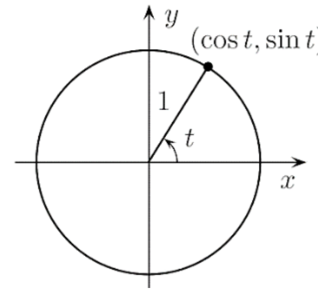
$\theta = \frac{-3\pi}{2}$



$\sin(\theta) =$ $\quad$ $\cos(\theta) =$ $\quad$ $\tan(\theta) =$

$\csc(\theta) =$ $\quad$ $\sec(\theta) =$ $\quad$ $\cot(\theta) =$

4.2 Day 2 – Domain and Period of Sine and Cosine



On the unit circle, what is the domain and range for the sine and cosine functions?

$\sin(t)$ $\cos(t)$

Situation 1	Situation 2
$\sin(\pi/6) =$ $\sin(\frac{\pi}{6} + 2\pi) =$	$\sin(30°) =$ $\cos(30°) =$ $\sin(-30°) =$ $\cos(-30°) =$
Adding 2π gets you back to the same spot, so $\sin(t + 2\pi n) =$ $\cos(t + 2\pi n) =$	Recall from a prior section that a function f is even when *f(-t) = f(t)* and it is odd when *f(-t) = -f(-t).*
Periodic: A function *f* is periodic if there exists a real number *c* such that $f(t + c) =$_____ We call *c* the period. What is the period for sine and cosine?	Even and Odd Trig Functions Cosine and secant are even functions. $\cos(-t) =$ $\sec(-t) =$ Sine, cosecant, tangent, and cotangent are odd functions. $\sin(-t) =$ $\csc(-t) =$ $\tan(-t) =$ $\cot(-t) =$

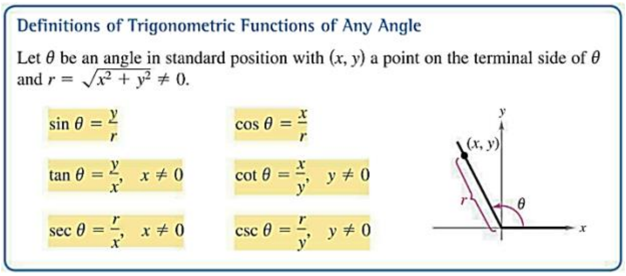
Example: Use the periodicity of sine and cosine to determine the following without a calculator.

a) $cos\left(\frac{13\pi}{6}\right)$	b) $sin\left(\frac{-7\pi}{2}\right)$

Example: Use a calculator and the reciprocal identities to determine the following, if possible. Round to four decimal places if necessary.

$csc\left(\frac{5\pi}{7}\right)$	$sec\left(-\frac{\pi}{2}\right)$	$cot(\pi)$

4.4 Trig Functions for Any Angle



Definitions of Trigonometric Functions of Any Angle

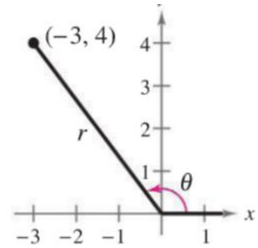
Let θ be an angle in standard position with (x, y) a point on the terminal side of θ and $r = \sqrt{x^2 + y^2} \neq 0$.

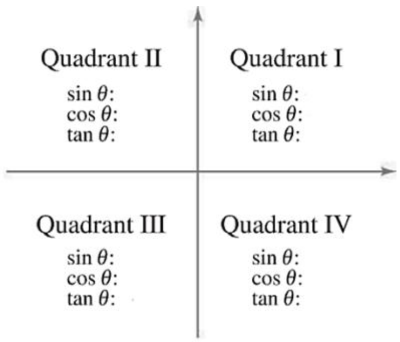
$\sin\theta = \frac{y}{r}$ $\qquad$ $\cos\theta = \frac{x}{r}$

$\tan\theta = \frac{y}{x},\ x \neq 0$ $\qquad$ $\cot\theta = \frac{x}{y},\ y \neq 0$

$\sec\theta = \frac{r}{x},\ x \neq 0$ $\qquad$ $\csc\theta = \frac{r}{y},\ y \neq 0$

Find sine, cosine, and tangent of θ.





State the quadrant θ lies for each.

a) $\sin(\theta) < 0$ and $\tan(\theta) > 0$

b) $\sec(\theta) > 0$ and $\cot(\theta) < 0$

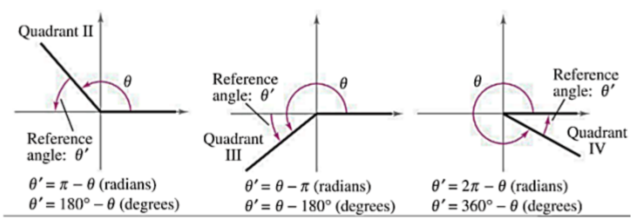
For each, find the indicated trig value in the specified quadrant.

Function	Quadrant	Trig Value Desired
a) sin(θ) = -3/5	IV	cos(θ)
b) sec(θ) = -9/4	III	tan(θ)

Definition of Reference Angle

Let θ be an angle in standard position. Its **reference angle** is the acute angle θ' formed by the terminal side of θ and the horizontal axis.

The reference angles for θ in Quadrants II, III, and IV are shown below.



Evlaute each.

a) $\sin\left(\frac{4\pi}{3}\right)$

b) $\tan(210°)$

c) $\sec\left(\frac{11\pi}{4}\right)$

Find the two solutions for each equation and give your answer both in degrees ($0° \leq \theta \leq 360°$) and radians ($0 \leq \theta \leq 2\pi$)

a) $\cos(\theta) = -1/2$

b) $\tan(\theta) = \frac{1}{\sqrt{3}}$