Honors Precalculus Name: ____________________________________

Chapter 2 Notes

Review of Factoring Quadratics and Completing the Square

Lead-In: Recall how to multiply (two techniques shown)

Multiple Distribution	Table
$(x+3)(x-5)$	$(x+3)(x-5)$

Factor x^2+bx+c and ax^2+bx+c

$x^2+4x-12$	$x^2-20x+42$
Factoring with fractions $3x^2+16x+5$	Factoring mentally $3x^2+16x+5$

$8x^2-10x+3$	$2x^2-9x+10$	$4x^2+7x-2$

Solve with factoring $2x^2-5x-3=0$	Solve by square roots $2x^2-98=0$

Solve by completing the square

$x^2+6x-20=0$

x^2	

$=20$

$x^2+9x+5=0$

x^2	

$=-5$

2.1 Day 1 – Write and Graph in Vertex Form

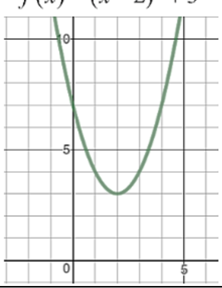
Lead – In: Use the Desmos Investigation to determine the effect of each parameter for Vertex Form:

$$f(x) = a(x-h)^2 + k$$

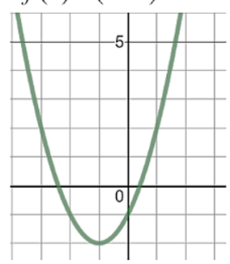
Parameter	Transformation to Parent Function $f(x) = x^2$
a	
h	
k	

Example: State the vertex for each graph. Do you see how it relates to its equation?

$f(x) = (x-2)^2 + 3$



$f(x) = (x+1)^2 - 2$



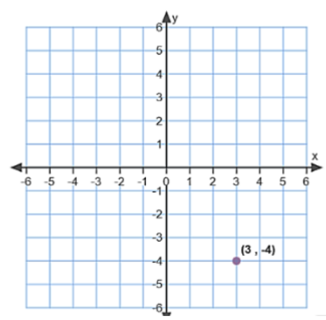
How to use "*a*" to Graph Outside of the Vertex

$g(x) = 2(x-3)^2 - 4$

x	
$g(x)$	

1) We know the vertex is located at (3,-4). What are two other values we could easily plug in for x to get two other points?

2) What is a shortcut way to use "a" to get two other points quickly, without having to plug anything in?



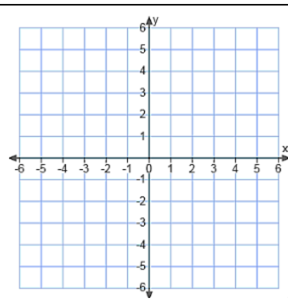
Steps to Graph in Vertex Form:

1)

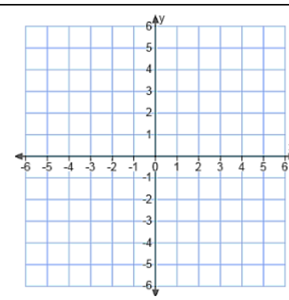
2)

3)

$f(x) = -3(x-5)^2 + 4$



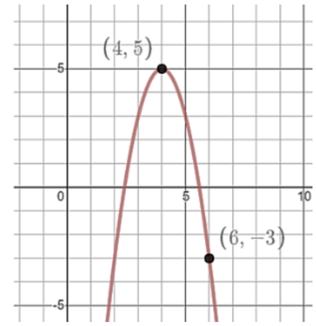
$f(x) = 0.25(x+2)^2 - 1$



<u>Example</u>: Working Backwards. Find the equation if given the graph.

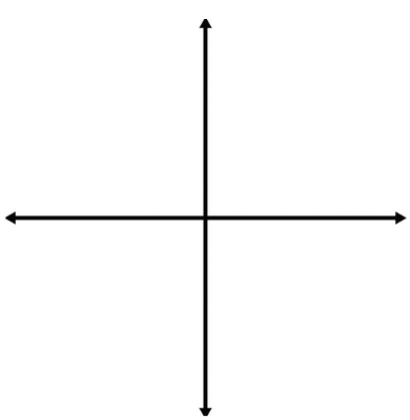
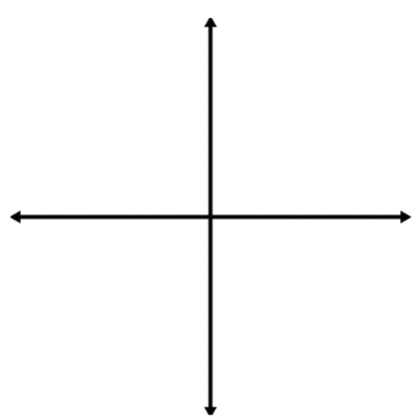
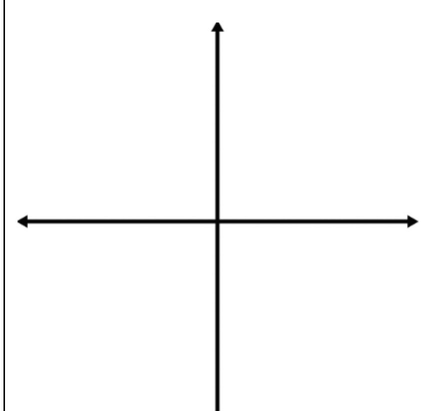
1) Use vertex

2) Plug in other point to solve for *a*



3) Write full equation

<u>Three Forms for Writing and Graphing a Quadratic Function:</u>

<u>Standard Form</u> $f(x) = ax^2 + bx + c$	**<u>Factored From</u>** $f(x) = a(x - m)(x - n)$	**<u>Vertex Form</u>** $f(x) = a(x - h)^2 + k$
- Tells you the y-int (c) - "a" tells you how the graph opens	- Tells you the factors (x-intercepts) at m and n. - "a" tells you how the graph opens	- Tells you the vertex is at (h, k) - "a" tells you how the graph opens
<u>Example</u>	**<u>Example</u>**	**<u>Example</u>**
<u>Key Info:</u>	<u>Key Info:</u>	<u>Key Info:</u>
Graph: 	Graph: 	Graph: 

2.1 Day 2 – Converting Between Three Forms for Quadratics

Standard $f(x) = ax^2 + bx + c$ **Factored** $f(x) = a(x-m)(x-n)$ **Vertex** $f(x) = a(x-h)^2 + k$

Factor

Convert $f(x) = 2x^2 + 7x + 3$ to factored form.

Standard $f(x) = ax^2 + bx + c$ **Factored** $f(x) = a(x-m)(x-n)$ **Vertex** $f(x) = a(x-h)^2 + k$

Complete the square

Convert $f(x) = 2x^2 + 8x + 7$ to vertex form.

Takeaway: For a quadratic function f(x) = ax^2 + bx + c, the vertex will be found at

Example: Find the vertex of $f(x) = 2x^2 + 8x + 7$ using the above result.

Standard $f(x) = ax^2 + bx + c$ **Factored** $f(x) = a(x-m)(x-n)$ **Vertex** $f(x) = a(x-h)^2 + k$

Expand by multiplying

Convert $f(x) = 2(x-3)^2 - 8$ to standard form and state the y-intercept.

Take the function *f(x) = 2x^2 - 5x - 3* and use a graphing calculator's CALC functions: *value (when x = 0), zero, and minimum or maximum* to determine the following.

- y-intercept
- x-intercepts
- vertex

According to physics, the path of a projectile moving through the air follows the path traced by the equation *h(t) = -16t^2 + vt + s*
t = time (sec)
h(t) = height at time t (ft)
v = initial velocity upwards (ft/s)
s = starting height (ft)
If a model rocket is launched with a takeoff speed of 200 ft/s from a 5 ft platform,
a) Write the equation

b) the maximum height

c) how long the rocket was in the air (no parachute)

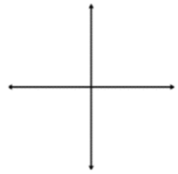
2.2 Day 1 – End Behavior and Zeros

Lead-In: Sketch a graph for f(x) and g(x) and then find the values.

$f(x) = x^2$

$f(1000) =$

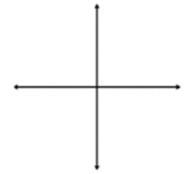
$f(-1000) =$



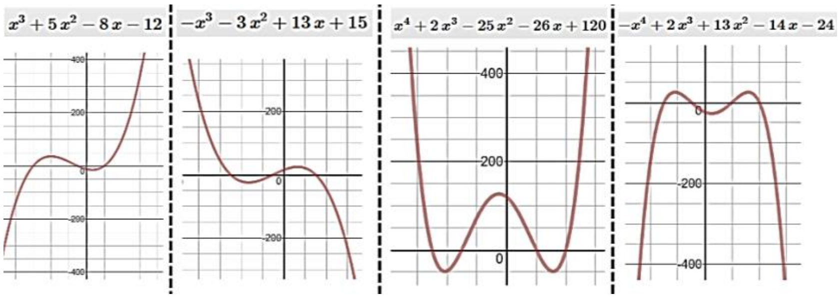
$g(x) = x^2$

$g(1000) =$

$g(-1000) =$



There are 4 graphs shown below. What do you notice?



Leading Term: $a_n x^n$

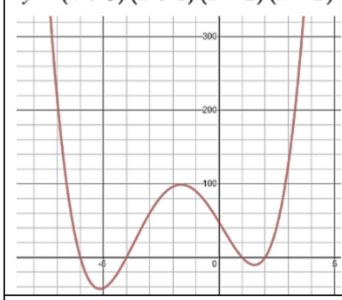
	Odd n	Even n
$a_n > 0$		
$a_n < 0$		

Example: Use the leading coefficient of each to determine the end behavior.

$f(x) = x^4 - 4x^2$
Leading Term = ________
Even or Odd: _______
End Behavior:

$f(x) = -2x^3(x-1)(x+1)$
Leading Term = ________
Even or Odd: _______
End Behavior:

$y = (x+6)(x+4)(x-1)(x-2)$



What do you notice?

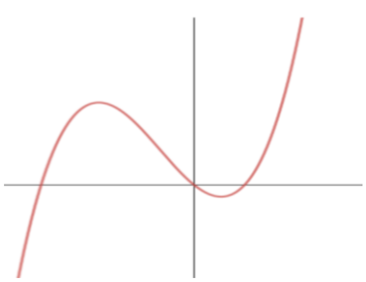
Real Zeros of Polynomial Functions

When f is a polynomial function and a is a real number, the following statements are equivalent.

1. $x = a$ is a *zero* of the function f.
2. $x = a$ is a *solution* of the polynomial equation $f(x) = 0$.
3. $(x - a)$ is a *factor* of the polynomial $f(x)$.
4. $(a, 0)$ is an *x-intercept* of the graph of f.

} These all imply the same thing

Ex: Find all the real zeros of $f(x) = x^3 + 2x^2 - 3x$ and mark the x-intercepts on the graph shown.



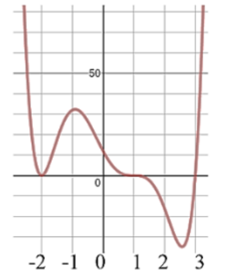
Ex: For the function $f(x) = 2x^2 + 12x + 8$, find the zeros (a) algebraically and (b) through a graphing utility.

a)

b)

2.2 Day 2 – Multiplicity, Writing and Graphing Polynomial Equations

$y=(x-3)(x+2)^2(x-1)^3$



What do you notice?

Multiplicity:

Degree	Action
1	
2	
3	
Even	
Odd	

Steps for Graphing a Polynomial

1) Factor, if not already in factored form.

2) Plot the x-intercepts and y-intercept (constant term).

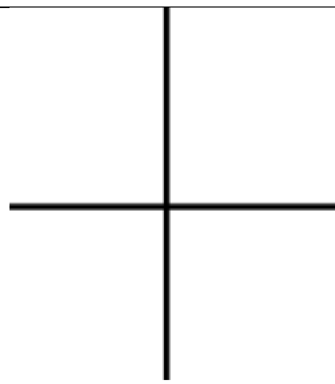
3) Sketch in the end behavior.

4) Use the multiplicity of each term to determine behavior at x-intercepts.

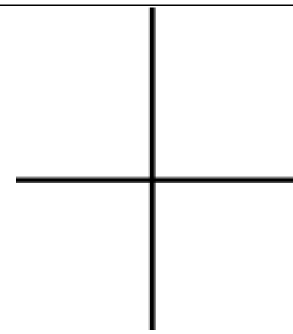
5) Sketch picture, plotting a few additional points if necessary. Good to consider points between known x-intercepts.

Examples

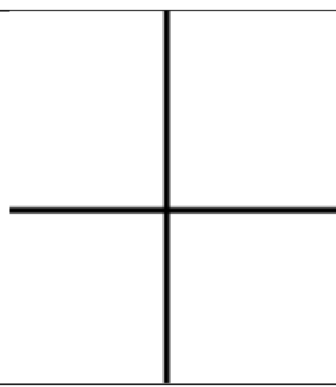
$y=-2(x-1)^2(x+2)$



$y=(x-2)^2(x+1)^2(x+3)$

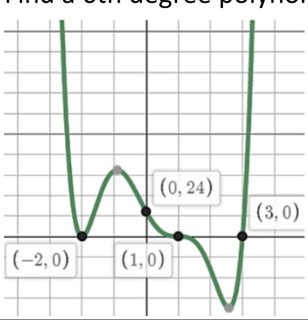


$y=-2x^3+6x^2-\frac{9}{2}x$



Find a polynomial function with zeros 4 (multiplicity 2) and -1/2.

Find a 6th degree polynomial for the graph shown.



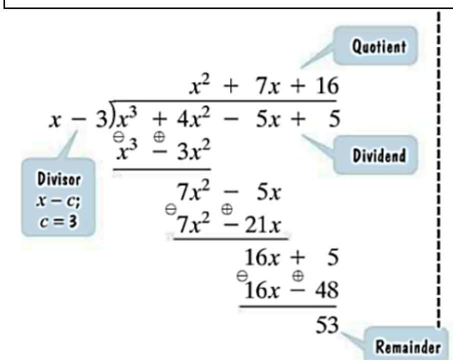
Zeros			
Multiplicity			
Factor			

How many solutions are there? Is the above the only one?

2.3 Day 1 – Division of Polynomials

Example: Similar to how we can do long division with numbers, we can do it with polynomials as well.

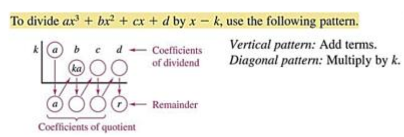
$(3x^3 + 2x^2 - 19x + 6) \div (x + 3)$	$(x^2 + 4x + 12) \div (x + 3)$
Verify our answer is correct.	$(2x^3 + 2x^2 - 4) \div (x - 1)$



3	1	4	−5	5
		3	21	48
	1	7	16	53

What do you notice? How are they similar?

***Synthetic Division**:*



If $x = -2$ is an x-intercept, use Synthetic Division to find the other intercepts for the graph of $f(x) = x^3 - 7x - 6$	Use synthetic division to divide even with a remainder. $(5x^3 + 6x + 8) \div (x + 2)$

2.3 Day 2 – Factor Theorem, Rational Root Test, and Finding All Zeros

Factor Theorem: a polynomial *f(x)* has a factor *(x – k)* if and only if: _________________

Example: Show that *(x-2)* is a factor of $f(x) = 2x^4 + 7x^3 - 4x^2 - 27x - 18$ by two different ways.

How do you find x-intercepts when you're not given one to start? Begin by trying to recognize the pattern below

$$x = \frac{3}{2} \qquad x = -\frac{1}{3}$$

$$0 = (2x-3)(3x+1)$$

$$0 = 6x^2 - 7x - 3$$

$$y = 6x^2 - 7x - 3$$

Rational Zero Theorem/Test

If $f(x) = a_n x^n + a_{n-1}x^{n-1} + \cdots + a_1 x + a_0$ has *integer* coefficients *then*

$$Possible\ Rational\ Zeros =$$

Example: *Use the Rational Zero Theorem to state all possible zeros in #1 and find all of the zeros in #2.*

$f(x) = 15x^3 + 14x^2 - 3x - 2$

Constant Term: ______

Leading Coefficient: ____

$f(x) = x^3 + 2x^2 - 5x - 6$

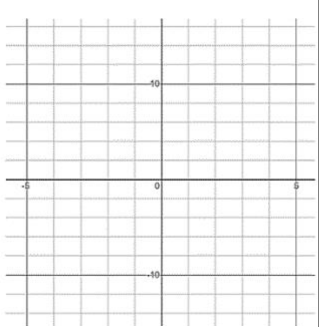
Constant Term: ______ *Leading Coefficient:* ___

Use the Rational Zero Theorem from our prior result to graph the function $f(x) = x^3 + 2x^2 - 5x - 6$

Zeros:

x-ints:

y -int:

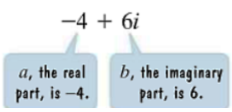


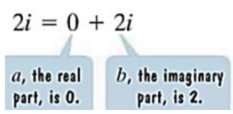
Find all the real zeros of $f(x) = x^4 - 5x^3 + 3x^2 + x$ and do not round (need exact answers).

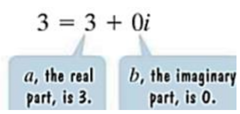
2.4 – Complex Solutions and Operations with Complex Numbers

Imaginary Unit (i): $i =$	**Principal Square Root**: $\sqrt{-a} =$
$2x^2 - 3 = -53$	$x^2 - 4x + 6 = 0$

Complex Number: number in the form $a + bi$ where *a* is the real part and *bi* is the imaginary part.







Example: add and subtract complex numbers (just like adding and subtracting like terms)

$(2 + 6i) + (12 - 3i)$	$(-3 + 4i) - (5i)$

Example: multiply complex numbers like you do when multiplying binomials (think multiple distribution or FOIL if it helps)

$(2 + 3i)(5 - 4i)$	$(5 + 2i)(5 - 2i)$

Divide Complex Numbers: When we divide complex numbers, we ensure there is only a real number in the denominator (what would it even mean to divide by 4 + 3i anyway?). We do this by multiplying the statement by the complex conjugate of the denominator.

Complex Conjugate:

$$(a + bi)(a - bi) = a^2 + b^2$$
$$(a - bi)(a + bi) = a^2 + b^2$$

Example: Divide the complex numbers.

Example: Challenge

$\dfrac{(2+5i)}{4+3i}$	$(3 + \sqrt{-5})(7 - \sqrt{-10})$

2.5 – Fundamental Theorem of Algebra and Finding all Zeros

$(x-4i)(x+4i)$	$(x+4i)(x+4i)$

Fundamental Theorem of Algebra: If *f(x)* is a polynomial of degree n (n>0), then *f* has ___________________________ in the complex number system.

Ex: For the polynomial $f(x) = x^3 + 16x$, a) state the maximum amount of zeros and then b) verify the values given are the zeros.	**Ex**: Find all the zeros of $f(x) = x^4 - 3x^3 + 6x^2 + 2x - 60$ given that (1 + 3i) is a zero of *f*.

Ex: Find all the zeros of $f(x) = x^5 + x^3 + 2x^2 - 12x + 8$

Ex: Find a third-degree polynomial that has zeros 2 and (1 – *i*) with a y-intercept of 12.

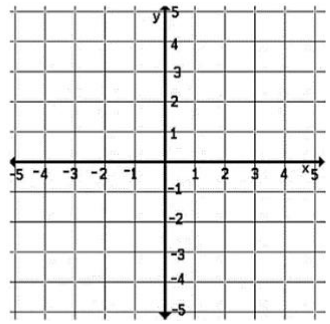
2.6 Asymptotes and Domain of Rational Functions

KeyConcept Rational Functions

Words	A rational function can be described by an equation of the form $y = \frac{p}{q}$, where p and q are polynomials and $q \neq 0$.
Parent function:	$f(x) = \frac{1}{x}$
Type of graph:	hyperbola
Domain:	$\{x \mid x \neq 0\}$
Range:	$\{y \mid y \neq 0\}$

x	y
-2	
-1	
-1/2	
-1/4	
0	
1/4	
1/2	
1	
2	

Lead-In: Graph the parent function $y = \frac{1}{x}$



Vertical Asymptote: the imaginary vertical lines (x equations) that occur where the domain is restricted. They are the values that make your denominator (but not numerator) ___________________________________.

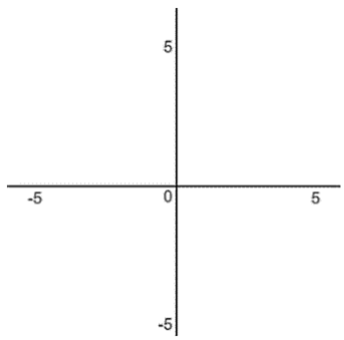
Example: State the domain, sketch in the vertical asymptote and use the graph of $y = \frac{1}{x}$ to sketch each.

$y = \frac{2}{x-3}$

Domain:

VA:

y-int:

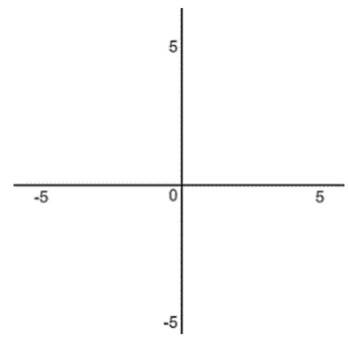


$y = \frac{-4}{x+1}$

Domain:

VA:

y-int:



Long-Run Behavior Intro

$$Say\ r(x) = \frac{x+3}{x+2}. \quad Question: \lim_{x \to \infty} \frac{x+3}{x+2}?$$

Note: $\frac{100+3}{100+2} =$ $\frac{10{,}000+3}{10{,}000+2} =$

As x gets large (goes to infinity), $r(x) \to$

Long-Run Behavior:

$\lim_{x \to \pm\infty} \frac{p(x)}{q(x)} =$

Horizontal Asymptote: $y = k$ is a horizontal asymptote of $\frac{p(x)}{q(x)}$ if

Example: State the horizontal asymptote of each graph, if there is one.

$y = \frac{3x}{x^2+5}$	$y = \frac{4x}{x-1}$	$y = \frac{4x^2}{x^2-1}$	$y = \frac{2x^3-5x}{x^2+3x-1}$

Zeros: x-values that make the output equal 0. For a rational function, when the ____________________________ = 0.

$y = \frac{x^2-x-12}{x^2+7}$	$y = \frac{3x}{x^2-10}$	$y = \frac{5}{x-3}$

Vertical Asymptotes: value(s) of x that make the ____________________ 0. These appear as vertical imaginary lines.

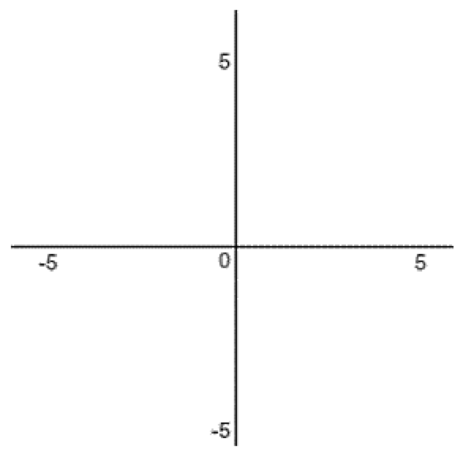
Horizontal Asymptotes: the value of y that your graph approaches as x gets very large. These appear as ________________ imaginary lines.

Zeros: x-values that ________________________, and therefore the output, 0. These show up in your graph as the x-intercepts.

Example: Put all this together and graph a rational function.

$f(x) = \frac{2x+4}{x-1}$ HA: VA:

Zeros: Y-int



Application Example: A pharmaceutical company wants to begin production of a new drug.

The fixed cost (research, testing, equipment) is $2,500,000.

On top of that, the drug costs $2,000 per gram to produce.

Total Cost Equation -> C(t) =

Why is it impractical for the company to produce small quantities? For example, evaluate and interpret C(10).

Can you define a function to calculate the average cost to produce q grams of the drug? Evaluate/interpret $\bar{C}(10{,}000)$.

What is the horizontal asymptote for the average cost function? What does it mean in real life?

2.7 Day 1 – Graphs of Rational Functions

Steps for Graphing Rational Functions

Step	**How**
1. Simplify $f(x)$	
2. Sketch H.A.	
3. Sketch V.A.	
4. Plot x-ints	
5. Plot y-int	
6. Other points	

Example: Apply the steps above and graph the following.

$f(x) = -\dfrac{2}{x-3}$

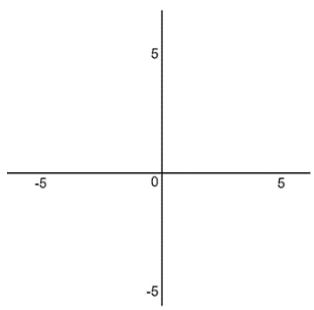
H.A:

V.A:

Zeros:

y-int:

x	2.99	3.01
y		



$f(x) = \dfrac{3x+6}{x+1}$

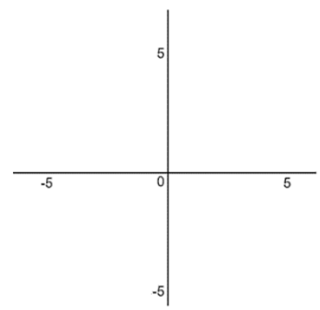
H.A:

V.A:

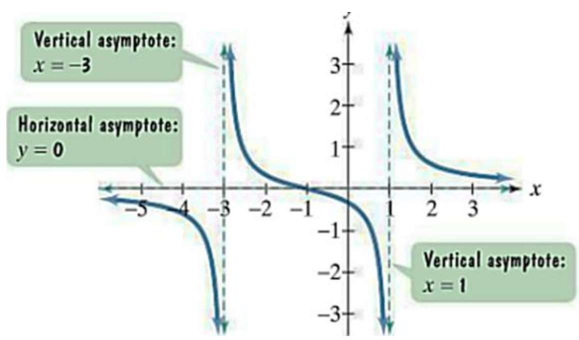
Zeros:

y-int:

x	-1.01	-0.99
y		



Example: Use the graph to fill in each statement.



As $x \to -3^-$, $f(x) \to$ ______.

As $x \to -3^+$, $f(x) \to$ ______.

As $x \to 1^-$, $f(x) \to$ ______.

As $x \to 1^+$, $f(x) \to$ ______.

As $x \to -\infty$, $f(x) \to$ ______.

As $x \to \infty$, $f(x) \to$ ______.

Example: Graph rational functions with multiple vertical asymptotes.

$y = \frac{x}{x^2-9}$

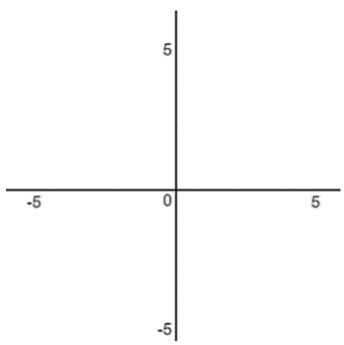
V.A:

H.A:

Zeros:

Y-Int:

x	-3.01	-2.99	2.99	3.01
y				



$y = \frac{3x^2}{x^2+x-6}$

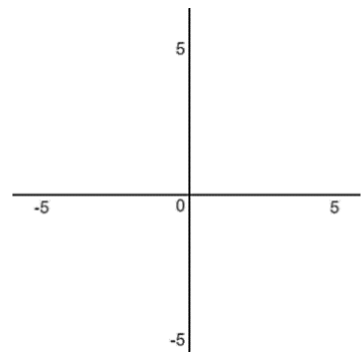
V.A:

H.A:

Zeros:

Y-Int:

x	-3.01	-2.99	1.99	2.01
y				



$y = \frac{x^2-9}{x^2+x-6}$

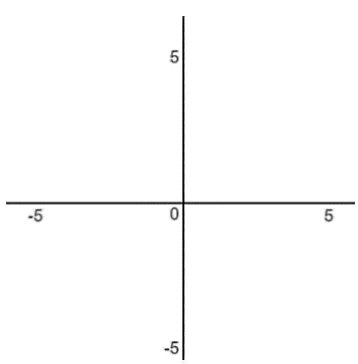
V.A:

H.A:

Zeros:

Y-Int:

x	1.99	2.01
y		



Ex: Work backwards

Write the possible formula for the rational function that:

-has a zero (x-int) at x = 2

-has VA at x = 5

-has HA at y = -3

2.7 Day 2 – Slant Asymptotes

For rational function, $f(x) = \frac{a_n x^n + a_{n-1}x^{n-1} + \cdots + a_0}{b_m x^m + b_{m-1}x^{m-1} + \cdots + b_0}$

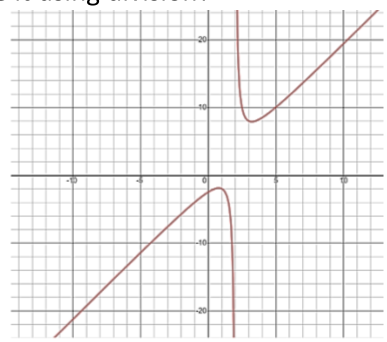
-If the degree of the denominator is greater than the degree of the numerator, the HA will be _______________.

-If the degree of the denominator is equal to the degree of the numerator, the HA will be________________.

-If the degree of the denominator is less than the degree of the numerator, the HA will ________________ and
____.

Example: What is the horizontal asymptote for the following? What if we rewrote it using division?

$f(x) = \frac{2x^2 - 5x + 5}{x - 2}$



Example: Graph each and always factor first if you can.

$f(x) = \frac{x^3}{2x^2 - 8}$

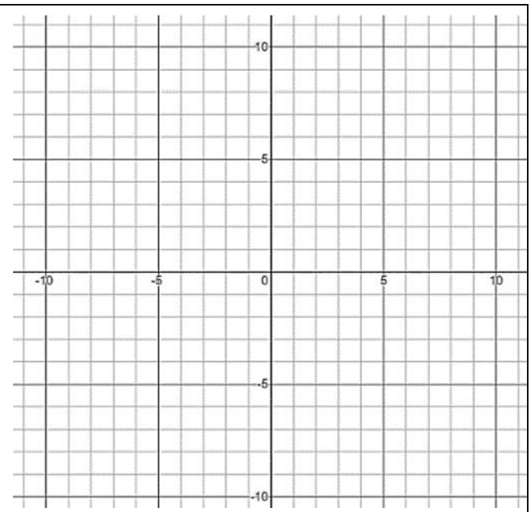
S.A:

V.A:

Zeros:

Y-Int:

x	-2.01	-1.99	1.99	2.01
y				



$f(x) = \frac{x^3}{x^2 + 4}$

S.A:

V.A:

Zeros:

Y-Int:

x	-6	-4	-2	2	4	6
y