Honors Precalculus Name: ____________________________________

Chapter 1 Notes

1.1 – Linear Equations

Decade	°F
1960s	57.18
1970s	57.20
1980s	57.52
1990s	57.76
2000s	58.12

Example: The following is data on the earth's average global temperature. How much has the temperature changed per year between 1960 and 2000?

Slope equation between two points (x_1, y_1) **and** (x_2, y_2)**:**

Lead-in to Modified Point-Slope Form: You are paid \$25 for each lawn you mow next summer. After mowing 8 lawns total, you now have \$275 in your bank account. If you were to mow 14 lawns total, how much will you have in your bank account?

How much would you have in your bank account after mowing 21 lawns total?

How much would you have in your bank account after mowing x lawns total?

Modified Point-Slope Form: To describe a line that goes through the point (x_1, y_1) with a slope of m, we write the following:

Example: Write the equation for the line with the following info.

$m = 2/3$ *and point*: $(4, -1)$	*points*: $(-2, 6)$ *and* $(-4, 12)$

Slope-Intercept Form:

Example: Write the equation for a line that goes through the points (-2, 7) and (6, 3). Two methods shown.

1) Write in point-slope first and simplify.	2) Write in y = mx + b, use the slope and coordinates of the point to solve for b.

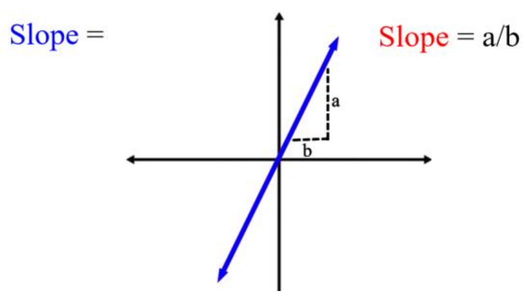
Parallel lines: two distinct nonvertical lines are parallel if and only if ____.

Using symbols we write:

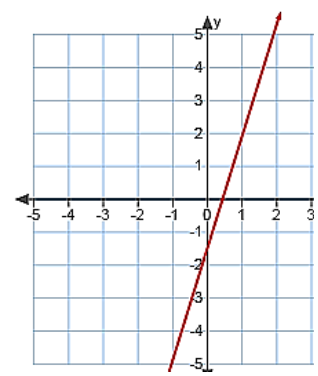
Perpendicular lines: two nonvertical lines are perpendicular if and only if their slopes are ______________________ ________________________.

Using symbols we write:

Why is this true for perpendicular lines?



Example: Write an equation of the line that passes through (-2,-4) and is parallel to the line y = 3x – 1. (Draw a picture if helpful)

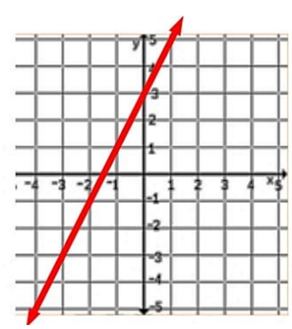


1) Identify slope.

2) Use point-slope first.

3) Simplify to slope-intercept.

Example: Write an equation of the line that passes through (4, -5) and is perpendicular to the line y = 2x + 3. (Draw a picture if helpful)



1) Identify slope.

2) Use point-slope first.

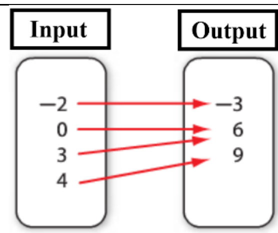
3) Simplify to slope-intercept.

1.2 - Functions, Domain and Range of Equations

Function:

Domain: **Range:**

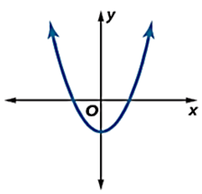
Input	1	3	5	1
Output	4	2	4	−4

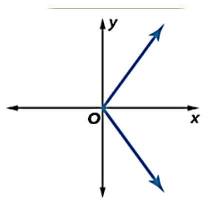


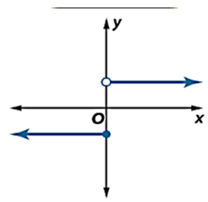
{(Helena, MT), (Boise, ID), (Butte, MT), (Pocatello, ID)}

Vertical Line Test:

Example: Use the vertical line test to determine if each graph is a function or not. Explain where the problem is.







Naming Functions – We use function notation: *f(input) = output* and *"f" is just the name of the function.*

Example: If $f(x) = 3x + 5$ and $g(x) = x^2 - 4$, find the following.

$f(2)$	$g(-6)$	*f(* ☺ *)*	$f(g(x))$

When determining the domain there are two restrictions: 1) You can't divide by ____ 2) Can't take square root of ___________

Example: State the domain of the following functions in set-builder notation

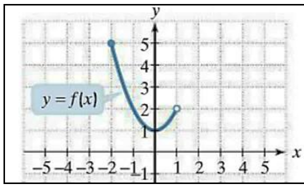
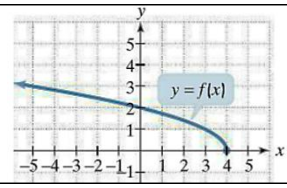
$\{x \mid x > 4\}$

"The set of all x such that x is greater than 4.

$f(x) = \frac{7}{x^2-25}$	$h(t) = \sqrt{t-9}$	$q(x) = \frac{7}{x^2+25}$

1.3 Day 1 – Domain & Range of Graphs and Key Features

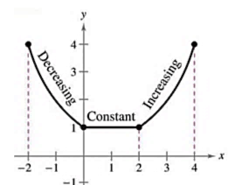
Example: for the graph of each function, state the domain, range, and *f(0).*

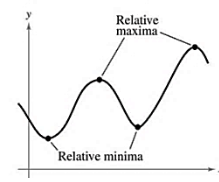
	D: R: $f(0) =$		D: R: $f(0) =$

Example: Find the domain and range of $f(x) = \sqrt{x-3}$

1) Graphically (use a calc)	2) Algebraically

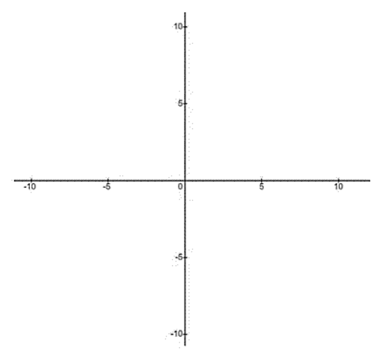
Key Features of a Graph: 1) intervals of increasing, decreasing, or constant 2) relative minimums or maximums

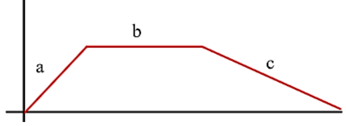
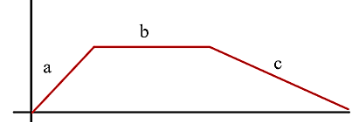




Example: Use a graphing calculator to sketch the graph, find the relative max/min, and state the open intervals of increase, decrease, or constant.

$f(x) = \frac{1}{2}x^3 - 2x^2$



Below is a graph of a car's distance from home over a period of time. Describe what is going on for each interval.	Below is a graph of a car's speed over a period of time. Describe what is going on for each interval.
 a) b) c)	 a) b) c)

1.3 Day 2 – Piecewise Functions and Odd/Even Functions

Piecewise Function:

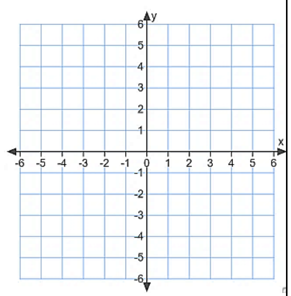
$$f(x) = \begin{cases} \text{equation 1, conditions for when equation 1 is true} \\ \text{equation 2, conditions for when equation 2 is true} \end{cases}$$

Tax rate	Single filers
10%	\$0 – \$9,700
12%	\$9,701 – \$39,475

Example: Write the piecewise function for the first two tax brackets for a single filer. Then use it to calculate how much federal tax you'd pay if you made \$30,000 in a year.

Example: Graph the piecewise function and evaluate.

$$f(x) = \begin{cases} 2x+3, x \le 1 \\ -x+2, x > 1 \end{cases}$$



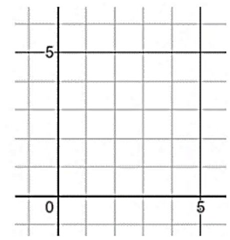
$f(3) =$

$f(0) =$

$f(1) =$

Write the equation and sketch the graph for $\lfloor x \rfloor$ from 0 to 4 on the x-axis.

$\lfloor x \rfloor =$

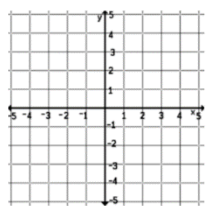


Make a table, graph each, and then fill in the blanks.

f(-2)= g(-2)=

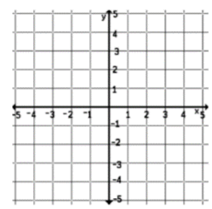
f(2)= g(2)=

$f(x) = x^2$



x	y
-2	
-1	
0	
1	
2	

$g(x) = x^3$



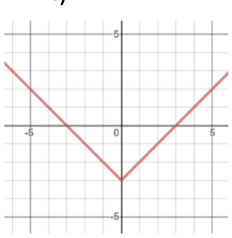
x	y
-2	
-1	
0	
1	
2	

Even Function: has the property that f(-x)=f(x) and the graph is symmetric w.r.t. ___________________

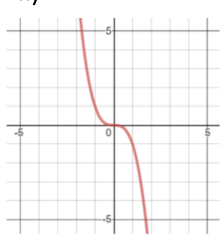
Odd Function: has the property that f(-x)=-f(x) and the graph is symmetric about ___________________

Example: Determine whether each function is odd, even, or neither. Explain.

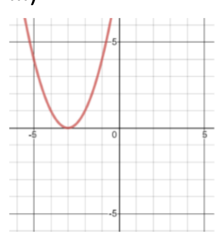
i)



ii)



iii)



i)

ii)

iii)

Example: determine whether the first function is indeed even and the second is odd.

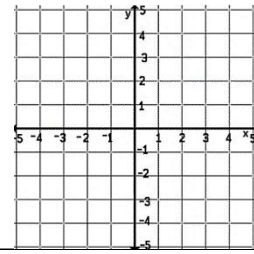
$f(x) = x^2 + 6$	$f(x) = x^5 + 1$

1.4 – Function Transformations

In this section we are going to learn how to transform (shift and scale) a function. If you know the parent function you can create an infinite amount of different transformations. There are a variety of parent functions we could use: linear, quadratic, cubic, square root, etc, however, for this portion we will use the absolute value function.

Example: Graph $f(x) = |x|$ by making a table.

x	y
-2	
-1	
0	
1	
2	

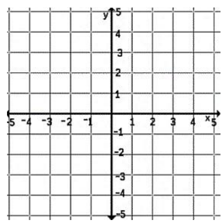


Grab your Chromebook, go to Desmos.com, and type in the equation. Then sketch the graph and fill in the table.

_______________________ Shift

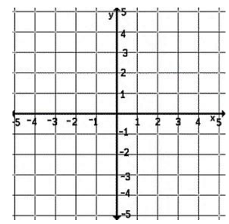
$y = |x| + 2$

x	y
-2	
-1	
0	
1	
2	



$y = |x| - 1$

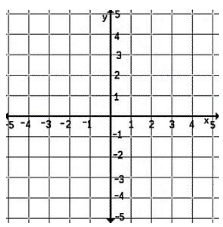
x	y
-2	
-1	
0	
1	
2	



_______________________ Stretch/Shrink

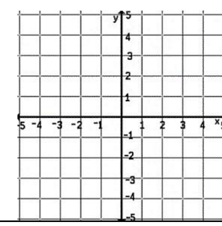
$y = 2|x|$

x	y
-2	
-1	
0	
1	
2	



$y = 1/2\ |x|$

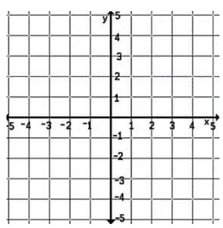
x	y
-2	
-1	
0	
1	
2	



_______________________ Reflections

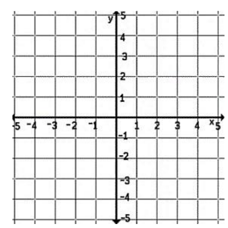
$y = -|x|$

x	y
-2	
-1	
0	
1	
2	



$y = |-x|$

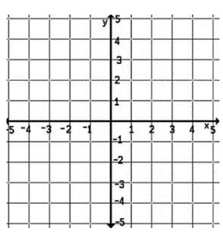
x	y
-2	
-1	
0	
1	
2	



_______________________ Shift

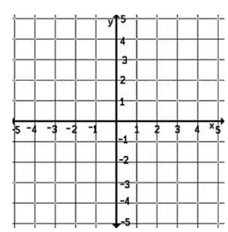
$y = |x - 1|$

x	y
-2	
-1	
0	
1	
2	



$y = |x + 1|$

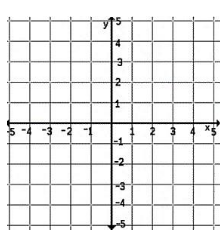
x	y
-2	
-1	
0	
1	
2	



_______________________ Stretch/Shrink

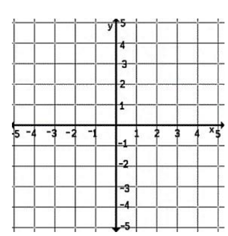
$y = |2x|$

x	y
-2	
-1	
0	
1	
2	



$y = \left|\frac{1}{2}x\right|$

x	y
-2	
-1	
0	
1	
2	



Example: Describe in words how the graph of g(x) compares to f(x) if $g(x) = 2f(x-3) + 7$

Example: If $f(x) = \sqrt{x}$ write an equation for $g(x)$ (<u>using the square root notation</u>) that would shift $f(x)$ to the right 3, be two times steeper, and up 9.

$g(x)$ = ________________________

<u>Combining Transformations</u>: when combining transformations, obey the order of operations and do the following in order: 1) horizontal shifting 2) stretching or shrinking 3) reflecting 4) vertical shifting

<u>Example:</u> Use the graph of f(x) to sketch $g(x) = -2f(x+2) - 1$. (Hint: use the table to help)

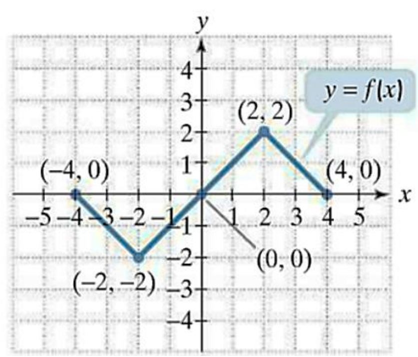
As you look at the following, let's first list all the transformations that should be applied (in order)

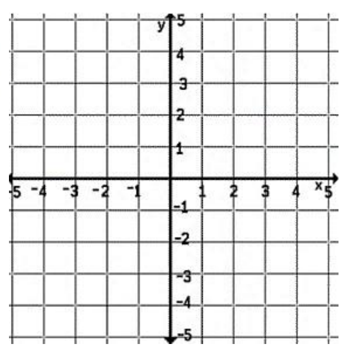
1. x-coordinates:

2. y-coordinates:

3. y-coordinates:

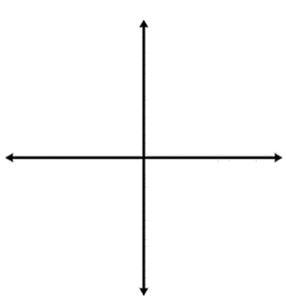
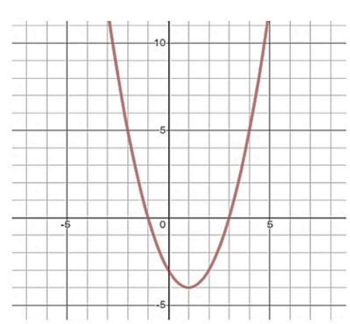
4. y-coordinates:





x	f(x)
-4	0
-2	-2
0	0
2	2
4	0

x	g(x)

Use the parent function to sketch a graph for $f(x) = -\sqrt{x+3} + 2$	Write the equation for the graph shown based on the parent function knowledge.
	

1.5 Day 1 – Combination of Functions

Operating with Functions – Just like we operate with numbers, we can operate with functions.

Sum: $(f+g)(x) = f(x) + g(x)$

Difference: $(f-g)(x) = f(x) - g(x)$

Product: $(fg)(x) = f(x) \cdot g(x)$

Quotient: $\left(\dfrac{f}{g}\right)(x) = \dfrac{f(x)}{g(x)}$, provided $g(x) \neq 0$.

Example: With $f(x) = x - 3$ and $g(x) = x^2 - 9$ find the following

$(f+g)(x)$	$(f-g)(x)$	$(fg)(x)$	$(\frac{f}{g})(x)$

Example: With $f(x) = x^2 - 1$ and $g(x) = x + 6$ given, find *(f + g)(5).*

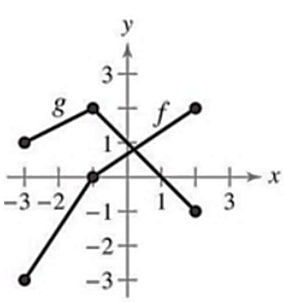
Add together first	Put 5 in first

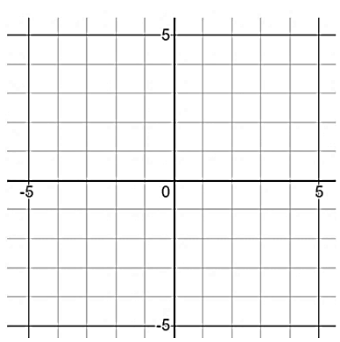
Example: If $f(x) = 1/x^2$ and $g(x) = 1/x$, find the following and state the domain.

$(\frac{f}{g})(x)$	$(\frac{f}{g})(t^2 - 4)$

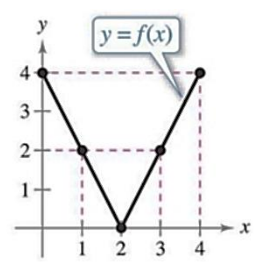
Example

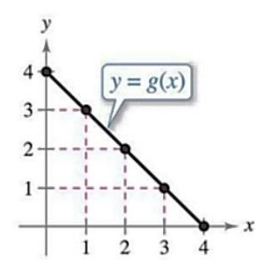
Use the graphs of f and g to graph *h(x) = (f + g)(x).*





Use the graphs of f and g to find *(f + g)(1)* and *(fg)(3).*





$(f+g)(1)$	$(fg)(3)$

1.5 Day 2 – Composition of Functions

Example: If *f(x) = x + 3*, evaluate the following.

$f(-4)$	$f(\pi)$
$f(t)$	$f($ 🎃 $)$

Composition of Functions:

Example: Evaluate the following with $f(x) = 2x - 1$ and $g(x) = x^2 + 3$.

$f(g(5))$	$g(f(4))$
$(f \circ f)(7)$	$(g \circ g)(7)$

Example: Evaluate the following with $f(x) = 3x + 1$ and $g(x) = x^2 - 9$.

$f(g(x))$	$g(f(x))$	$f(f(x))$

Example: Give the practical meaning of the composite function V(m(t)) where m(t) is the miles on a car that is t years old and V(x) is the value of a car that has x miles on it.

V(m(t)) represents:

Example: Is the following always true, sometimes true, or never true? $f \circ g = g \circ f$

1.6 Day 1 – Inverse Functions

Biologists have found that the number of cricket chirps per minute is a function of the outside temperature. The function is: *C(t) = 4(t-40).* Use this function to calculate the following and interpret them.

C(75)= C(40)=

Biologists also find it useful to be able to count the number of chirps and then use that to calculate an estimate for the temperature. In order to do so, one would need to solve the following equation for t.

Original: *C = 4(t-40)* ***Inverse:***

Inverse Function *(definition):* functions *f(t)* and $f^{-1}(t)$ are inverses if the __________ of *f* is the __________ of f^{-1} and the ___________ of *f* is the ____________ of f^{-1}. *So, if* $f(a) = b$ then .

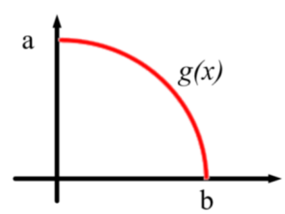
Example: Based on the cricket example, describe in words what the following represent:

C(45) = 20 $C^{-1}(80) = 60$

x	-2	0	3	5
g(x)	-1	8	7	3

$g(3) =$ $g^{-1}(7) =$

$g(0) =$ $g^{-1}(3) =$



$g(0) = ?$

$g(?) = 0$

$g^{-1}(?) = 0$

$g^{-1}(0) = ?$

Finding the Inverse Function Equation (!!!!NOTE: $f^{-1}(x) \neq \frac{1}{f(x)}$)

Example: If $f(x) = 3x + 4$ find $f^{-1}(x)$.

Example: Using your values from the last example, find:

$f(f^{-1}(x))$

$f^{-1}(f(x))$

Inverses undo one another so, if two functions are inverses then: ____

Example: determine if $f(x) = 5x - 9$ and $g(x) = \frac{x+5}{9}$ are inverses.

Show that $f(x) = 3x + 4$ and $g(x) = (x - 4)/3$ are inverses:

a) Algebraically.
b) Numerically.

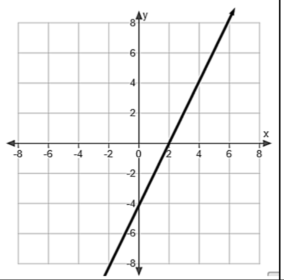
1.6 Day 2 – Inverse Graphs

Example: Graphing Inverse Functions

The table and graph for $f(x) = 2x - 4$ is shown below. Make the table and graph for its inverse.

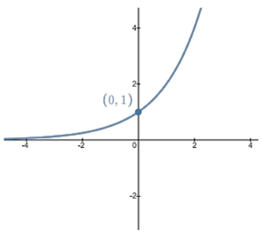
x	f(x)
-2	-8
-1	-6
0	-4
1	-2
2	0

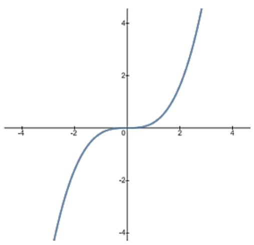
x	$f^{-1}(x)$



When you graph inverse functions, they are reflections of one another across the line ______________. Why is this so?

Example: Graph the inverse of each function.



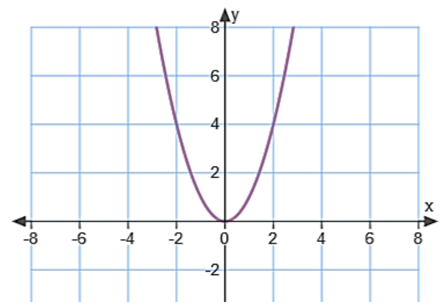


Example: The table and graph for $f(x) = x^2$ is shown below.

Make the table and graph for the inverse of $f(x) = x^2$.

x	f(x)
-2	4
-1	1
0	0
1	1
2	4

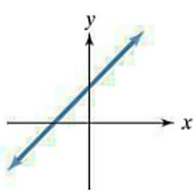
x	$f^{-1}(x)$

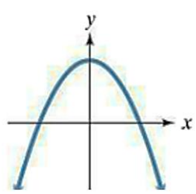


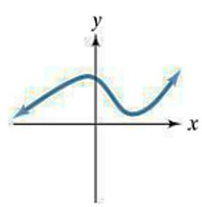
Where is the problem with this inverse?

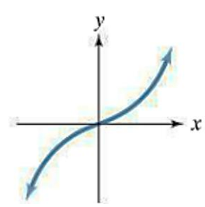
Horizontal Line Test: a function f has an inverse function, f^{-1}, if there is no ______________________ line that intersects the graph of f at more than one point.

Example: Which functions below will have an inverse function?

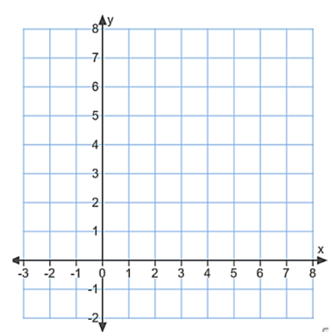








Example: We saw the graph for $f(x) = x^2$ fails the horizontal line test and is therefore not going to have an inverse function. How could we adjust the domain of f so it will have an inverse?



To see how a graphing calculator can graph an inverse relation for you (but still up to you to determine if the relation is a function!), you can go to: tinyurl.com/CHSgraphinverse

1.7 – Linear Regression

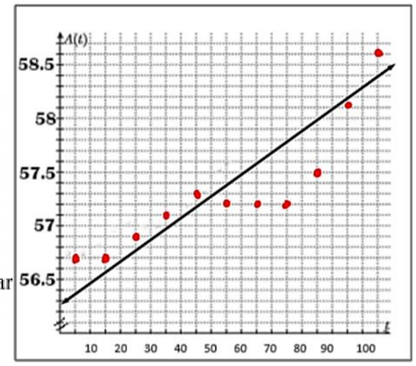
Way to Graph Paired Data: Scatter Plot: Used to determine the correlation (relation) between paired data.

Linear Regression: process of modeling a ______________________ between paired data with a linear model

Example: Scientists recorded Earth's average temperature over the 20th century and found the following.

t (years after 1900)	5	15	25	35	45	55	65	75	85	95	105
A (avg. temp in Fahrehneit)	56.73	56.73	56.89	57.13	57.27	57.17	57.17	57.2	57.53	58.14	58.60

To obtain the linear regression equation, follow the following steps.



Step 1 – Enter your Data

- Hit [STAT]
- Choose **1:Edit** by either hitting [1] or [ENTER] .
  If necessary, clear out any old data in the lists:
  Use [▲] to get cursor to cover L1 at top of list; press [CLEAR][ENTER]. Repeat process for L2.
- Type the data values for the independent (x) variable in column L1. Hit [ENTER] after each entry.
- When you finished entering data in L1, hit [▶] and then enter the data values for the dependent (y) var in column L2.

Step 2 – Getting the Equation for the Line of Best Fit

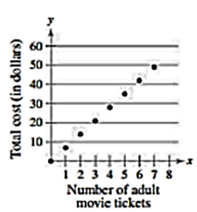
- Hit [STAT] then [▶] to CALC
- Choose 4:LinReg(ax+b) (Either scroll down to 4 and then hit [ENTER], or simply hit [4])

In the calculator the a-value represents the slope, the b-value represents the y-intercept, and r is the correlation coefficient.

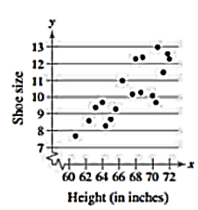
Linear Regression Equation: _________________________________

Correlation Coefficient (r-value): tells you how good of a linear fit you have. The closer the absolute value is to 1, the better the linear fit. If it is positive, the correlation is positive. Likewise, if the correlation coefficient is negative, the correlation is negative.

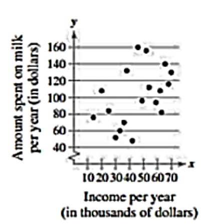
A correlation whose absolute value is greater than 0.8 is generally described as strong, whereas a correlation whose absolute value is less than 0.5 is generally described as weak.



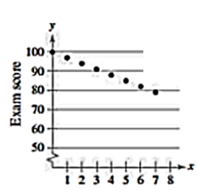
Perfect positive correlation
$r =$



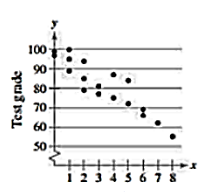
Strong positive correlation
$r =$



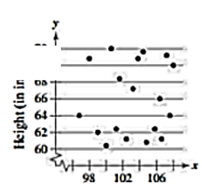
Weak positive correlation
$r =$



Perfect negative correlation
$r =$



Strong negative correlation
$r =$



No correlation
$r =$

Our r-value was a 0.89, so the relationship shows a ____.

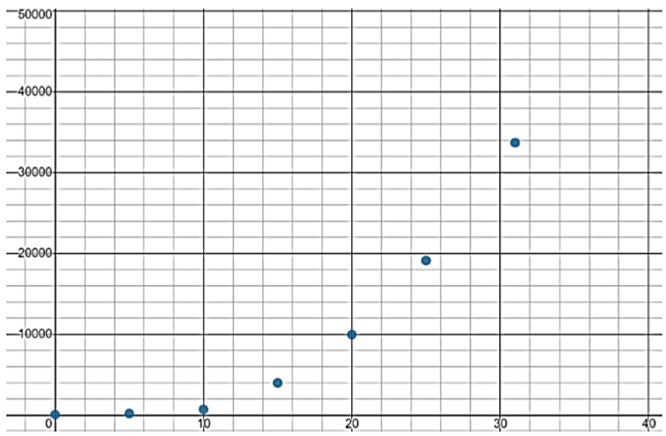
Interpreting the Linear Regression Equation

Our equation is y = 0.015x + 56.48. Use it to estimate the earth's average temperature in 2010.

Other Models and Regression Equations

Not all models are linear though. If you look at the amount of new daily COVID-19 cases for the US from March 4 until April 4 of 2020 it looks like this (March 4 is day 0).

x_2	y_2
0	26
5	149
10	662
15	3948
20	9939
25	19118
31	33725



What type of regression could we ask the calculator to do here?

Use the _______________________ regression equation of

to estimate the number of cases on March 30 (day 26). Then compare to the actual value given below.