Honors 1 Name: ___________________________________

12.1 – Samples and Studies

Statistics:

Population:

Sample:

Examples: In the following studies, state the population and the sample.

A researcher was interested in the effectiveness of a program in preparing high school juniors nationwide for the ACT so one school in each state was selected to participate in a test study.	A researcher is interested in the average number of hours of TV watched per week by children under the age of 5, so a sample of 100 MT children under the age of 5 is conducted using hospital birth records.	A high school principal is deciding whether the school should change its mascot so she decides to survey some of the students.

Bias:

Five Types of Samples

Simple Random Sample: each member of a population has an equal chance of being selected. Ex:	**Systematic Sample:** members are selected according to a specified interval from a random starting point. Ex:	**Self-Selected Sample**: members volunteer to be included in the sample. Ex:
Convenience Sample: members who are readily available or easy to reach are selected. Ex:	**Stratified Sample:** the population is first divided into similar, nonoverlapping groups, and then members are randomly selected within the groups. Ex:	

Three Types of Studies

Type	Definition
survey	Data are collected from responses given by a sample regarding their characteristics, behaviors, or opinions.
observational study	Members of a sample are measured or observed without being affected by the study.
experiment	The sample is divided into two groups: • an *experimental group* that undergoes a change, and • a *control group* that does not undergo the change. The effect on the experimental group is then compared to the control group.

Determine the Type of Study

1) A company shows 5 different commercials to a group of students and records their reactions.

2) Scientists study the behavior of two groups of rats (sugar and no sugar) to determine sugar's effect on their ability to complete a maze.

3) The school board is interested in parents' thoughts on building new schools in the district so a questionnaire is sent home in the newsletter.

Determine whether the following are biased or unbiased:

1) Do you like animals, and would you ever consider having a dog or cat as a pet?

2) What type of music do you listen to?

3) To study the effects of a new training method a dog trainer tries the old method on a control group of terriers and the new method on an experimental group of retrievers.

12.2 – Statistics and Parameters

Parameter: measure that describes a characteristic of a population. This is the value we want to know but usually the population is so large we can't calculate it.

Statistic: measure that describes a characteristic of a sample, which is being used to estimate the parameter.

Example: At a local university, a random sample of 40 scholarship applications is selected. The mean GPA of the 40 applicants is calculated.

Sample:

Population:

Sample Statistic:

Population Parameter:

Example: An NFL team is debating between two running backs to draft for their team. The number of rushing yards from the past 5 games is shown below. Which one should they pick? What should be calculate to decide?

Option A: 75, 80, 85, 90, 85

Option B: 40, 120, 40, 60, 160

Standard Deviation:

KeyConcept Standard Deviation

Step 1 Find the mean, $\bar{x}$.

Step 2 Find the square of the difference between each data value x_n and the mean, $(\bar{x} - x_n)^2$.

Step 3 Find the sum of all of the values in Step 2.

Step 4 Divide the sum by the number of values in the set of data n. This value is the variance.

Step 5 Take the square root of the variance.

Formula $\sigma = \sqrt{\dfrac{(\bar{x} - x_1)^2 + (\bar{x} - x_2)^2 + \ldots + (\bar{x} - x_n)^2}{n}}$

St. Dev A =

St. Dev B =

Conclusion:

Use a graphing calculator to find the mean and standard deviation. Clear all lists. Then press STAT ENTER, and enter each data value into L1. To view the statistics, press STAT ▶ 1 ENTER.

Example: A soda company is choosing between two machines that will fill up their cans. They take a sample of 5 cans filled up by both machines. Each machine is attempting to fill up the cans to 12 oz. Which should they choose?

Machine 1: 12.1, 11.8, 11.9, 12, 12.1

Machine 2: 12.2, 12.3, 12.3, 12.3, 12.2

Mean =

St. Dev =

Mean =

St. Dev =

12.3/12.4 – Box Plots and Histograms

Example: The prices of 11 houses in the Helena area (in thousands) is shown: 135, 150, 180, 200, 225, 230, 250, 270, 300, 350, 500. The **five-number summary** are the following five values of a data set:

Minimum: Smallest value =

Lower Quartile (Q1): middle number between minimum and median =

Median: Middle value in data set =

Upper Quartile (Q3): middle number between median and maximum =

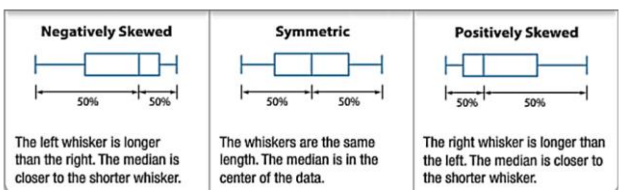
Maximum: Largest value =

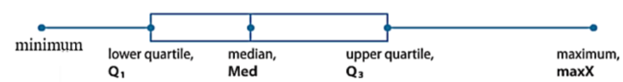
*Note: You can have a calculator provide these if you follow the same steps as outlined in 12.2 and scroll down.

Example: Have a calculator compute the mean and median if we add one more house: 1500 (1.5 million).

Before	After

Boxplot:





Enter the data as L1. Press 2nd [STAT PLOT] ENTER ENTER and choose ⊡. Next, press zoom and choose option 9: 9↓ZoomStat

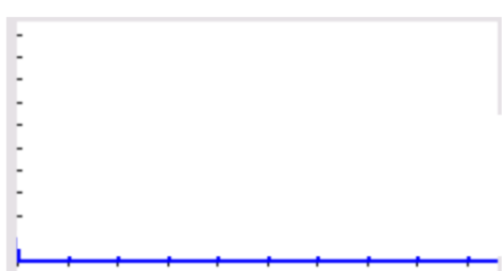
Example: Construct a box plot for the house data (without the $1.5 million house).

Histogram:

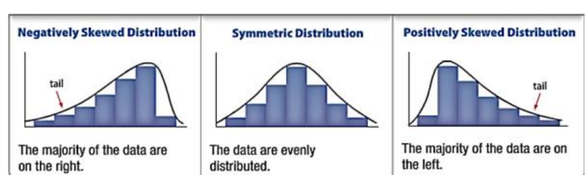
Example: Construct a histogram for a study of 17 students who got the following # of hours of sleep:

4, 6, 6, 6, 6, 6, 7, 7, 7, 7, 7.5, 7.5, 8, 8, 9, 10, 12

hours	frequency
4-4.9	
6-6.9	
7-7.9	
8-8.9	
9-9.9	
10-10.9	
12-12.9	



Hours of Sleep Each Night



Enter the data as L1. Press 2nd [STAT PLOT] ENTER ENTER and choose ⊓. Next, press zoom and choose option 9: 9↓ZoomStat

12.6 Day 1 – Factorials and Permutations

Permutation:

Example: How many different ways can a red, blue and green book be arranged on a shelf?

Option 1: Make the sample space (list all possibilities)	Option 2:

Factorial (# of ways to rearrange *n* items):

1! = 7! = 0! $\frac{6!}{4!} =$

Examples

1) A student tells his friend the password for his phone uses the numbers 5, 3, 2, 1. How many combinations are possible?

2) A travel agency offers an Italian package for visits to 5 cities. The customer can pick the order of the cities. How many different packages are possible?

Example: What if in the Italian vacation package they offered 5 cities and you could only visit 3 of them, but you still pick the order? How many different packages would be available now?

Permutation: a specific arrangement of *r* items from a set of *n* objects

KeyConcept Permutation Formula

Words The number of permutations of *n* objects taken *r* at a time is the quotient of *n*! and $(n - r)!$.

Symbols $P(n, r) = \frac{n!}{(n-r)!}$

Example: Use the equation to calculate P(6,3) and P(4,1).

Examples

1) A soccer team has 11 players. How many ways can a coach select the order of 5 players to take penalty kicks for a tied game?

2) A computer program asks you to make a 7 character-long password using numbers and letters, but no character can repeat. How many different combinations are possible?

12.6 Day 2 – Combinations and Deciding How to Count

Lead-In: A baseball coach is deciding between 5 players for his top 3 spots in the batting order. How many different orders are there?

However, what if he doesn't care what order they are in, rather he just cares that a certain 3 were selected? So, how many different ways could each selection of 3 players have been chosen?

Each combination of 3 players could be arranged in 3! = 6 different ways. Therefore, out of the 60 possible lineups, there are _____ = ____ different combinations.

Combination: any arrangement of *r* items from a group of *n* objects

*Note: Why is $C(n,r) \leq P(n,r)$?

KeyConcept Combination Formula

Words	The number of combinations of *n* objects taken *r* at a time is the quotient of $n!$ and $(n-r)!r!$.
Symbols	$C(n,r) = \frac{n!}{(n-r)!r!}$

Note: $C(n,r) = \frac{P(n,r)}{r!}$

Examples

15 companies have applied for commercial time slots during the Super Bowl. How many different sets of companies can be selected if only 10 slots are available?	For a court case there are 30 prospective jurors. How many different ways can a panel of 12 jurors be selected?

When to Use Permutation? When to Use Combination?

-We use combinations when the order of the selection does not matter.

-We use permutations when the order does matter.

Examples: Identify whether this is a combination or permutation and then state number of possibilities.

1) selecting 3 songs from your 13 song playlist	2) selecting 4 side menu options from 6 total	3) selecting a CEO and vice-CEO from 18 applicants

Recall: We agreed the number of combinations will never be greater than the number of permutations. But when are they equal?

12.7 – Probability of Compound Events

Lead-In: Mr. and Mrs. Smith have two children. What is the probability they had a boy and then had a girl?

Option 1) List all possibilities for 2-child families.	Option 2) Calculate it.

Compound Event:

Joint Probability:

Independent Events:

Dependent Events:

KeyConcept Probability of Independent Events

Words: If two events, A and B, are independent, then the probability of both events occurring is the product of the probability of A and the probability of B.

Symbols: $\boxed{P(A \text{ and } B) = P(A) \cdot P(B)}$

KeyConcept Probability of Dependent Events

Words: If two events, A and B, are dependent, then the probability of both events occurring is the product of the probability of A and the probability of B after A occurs.

Symbols: $\boxed{P(A \text{ and } B) = P(A) \cdot P(B \text{ following } A)}$

Examples: State whether each is independent or dependent, then calculate the probability.

1) You roll two dice. Find the probability you roll a 6 and an odd number.	2) You draw 3 cards from a deck (without replacing). Find the probability of getting the 3 cards in this order. P(diamond, club, not a club)

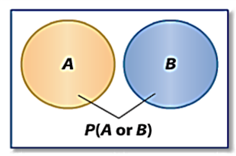
Mutually Exclusive:

KeyConcept Probability of Mutually Exclusive Events

Words: If two events, A and B, are mutually exclusive, then the probability that either A or B occurs is the sum of their probabilities.

Symbols: $P(A \text{ or } B) = P(A) + P(B)$

Model



Examples: Monopoly/rolling dice questions

1) What is the probability you roll a 3 or more?

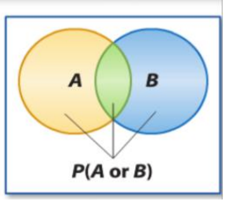
2) What is the probability you roll a 3 or a 5?

KeyConcept Probability of Events that are Not Mutually Exclusive

Words: If two events, A and B, are not mutually exclusive, then the probability that either A or B occurs is the sum of their probabilities decreased by the probability of both occurring.

Symbols: $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$

Model



Examples: You draw a card from a deck. State whether each is mutually exclusive or not and then calculate.

P(7 or a jack)

P(spade or a nine)

12.8 – Probability Distributions

A gaming software company with 5 online games on the market is interested in how many games each of their customers play so they surveyed 800 randomly chosen customers.

1) Find the probability that a randomly chosen customer plays 3 games.

2) Find the probability that a randomly chosen customer plays at least 4 games.

Number of Computer Games	Number of Customers
1	130
2	110
3	150
4	300
5	110

Probability Distribution:

Probability Graph:

*Requirements for a probability distribution: 1) 2)

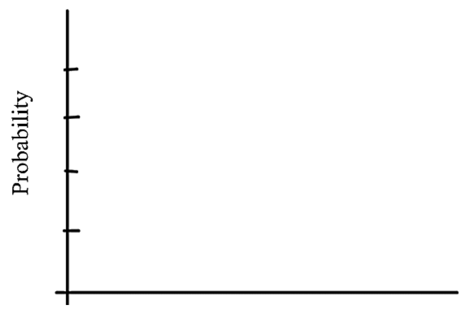
Example: The probability distribution for the blood types of the general population is shown.

Blood Type	A	B	AB	O
Percentage	0.41	0.12	0.03	0.44

1) Show the distribution is valid.

2) What is the probability of the following? P(A or B) = P(not O) =

3) Make a probability graph.



Finding the Expected Value (Mean or Average)

Let's return to the first example of the video game company and calculate the average number of games played by a customer.

Method 1 (long way)

Method 2 (efficient way)

first make a probability distribution

Number of Computer Games	Probability
1	
2	
3	
4	
5	

KeyConcept Expected Value of a Discrete Random Variable

Words The expected value of a discrete random variable is the weighted average of the values of the variable. It is calculated by finding the sum of the products of every possible value of X and its associated probability $P(X)$.

Symbols $E(X) = [X_1 \cdot P(X_1)] + [X_2 \cdot P(X_2)] + \ldots + [X_n \cdot P(X_n)]$, where n is the total number of values of X

Expected Value with "1 in" Probabilities

An insurance company has collected the following data on the probability of an accident costing a certain amount and is wondering what to charge for their six month-policy.

Cost	Probability
\$1000	1 in 15
\$5000	1 in 50
\$10,000	1 in 150
\$20,000	1 in 400