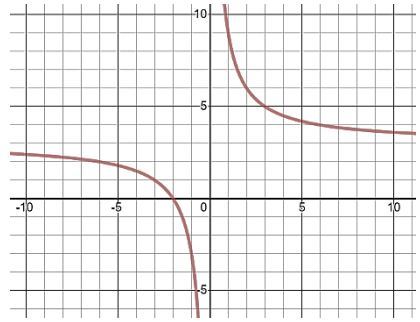
Honors Precalculus Name: _____

Chapter 11 Notes

11.1 Day 1 Limit Definition and Limits that Exist



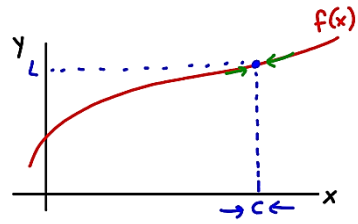
Lead-In

$f(x) = \frac{6}{x} + 3$

$\lim_{x\to\infty} \frac{6}{x} + 3 =$ $\lim_{x\to-\infty} \frac{6}{x} + 3 =$

$\lim_{x\to 0^+} \frac{6}{x} + 3 =$ $\lim_{x\to 0^-} \frac{6}{x} + 3 =$

Limit: if $f(x)$ becomes arbitrarily close to a unique number *L* as *x* approaches *c* from either side, then the **limit** of *f(x)* as *x* approaches *c* is *L*. Mathematically, we write this as $\lim_{x\to c} f(x) = L$



Ex: Fill in the table for $f(x) = x^2 - 5$ and use the table to determine the limit:

$\lim_{x\to 3} x^2 - 5 =$

x	2.9	2.99	2.999	3	3.001	3.01	3.1
f(x)							

Ex: Fill in the table for $f(x) = \frac{x-1}{x^2+3x-4}$ and use the table to determine the limit:

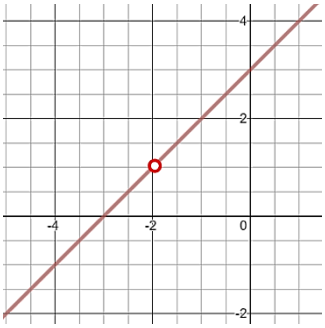
$\lim_{x\to 1} \frac{x-1}{x^2+3x-4} =$

x	0.9	0.99	0.999	1	1.001	1.01	1.1
f(x)							

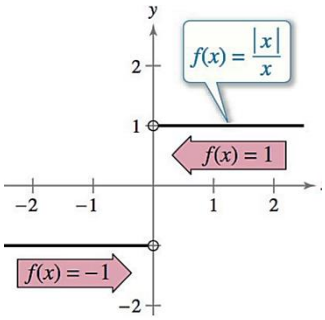
Note: What is $f(1)$ though?

Ex: Simplify the following rational function and then use its graph to determine the limit shown.

$\lim_{x\to -2} \frac{x^2+5x+6}{x+2} =$

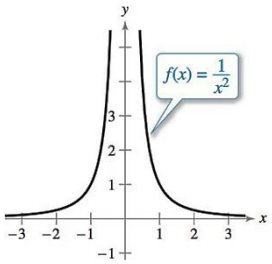


Ex: The graph for $f(x) = \frac{|x|}{x}$ is shown. Use it to show $\lim_{x\to 0} \frac{|x|}{x} = DNE$.



11.1 Day 2 Limits that Do Not Exist and 11.2 Techniques for Evaluating

Explain why the limit DNE for $\lim_{x\to 0} \frac{1}{x^2}$.

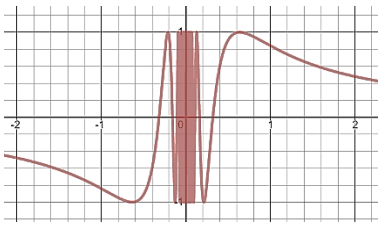


Determine whether the following limit exists.

$$\lim_{x\to 0} sin\left(\frac{1}{x}\right)$$

First, think about what happens to 1/x as $x \to 0$

Consider the graph.



Conditions Under Which Limits Do Not Exist

The limit of $f(x)$ as $x \to c$ does not exist under any of the following conditions.

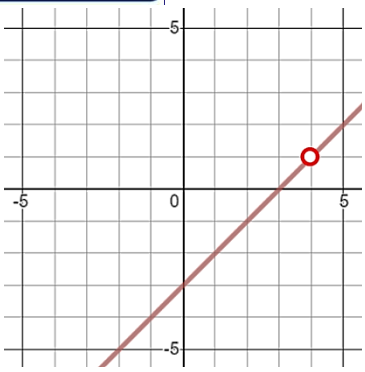
1. $f(x)$ approaches a different number from the right side of c than it approaches from the left side of c.
2. $f(x)$ increases or decreases without bound as x approaches c.
3. $f(x)$ oscillates between two fixed values as x approaches c.

Ex: Evaluating Limits by Dividing Out

$$\lim_{x\to 4} \frac{x^2-7x+12}{x-4}$$

Initially, if you try direct substitution, what happens?

If you graph it, you see the limit exists. How can you get it algebraically?

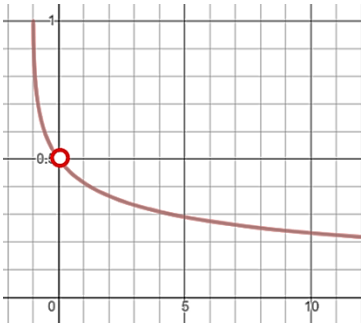


Ex: Evaluating Limits by Rationalizing

$$\lim_{x\to 0} \frac{\sqrt{x+1}-1}{x}$$

Initially, if you try direct substitution, what happens?

If you graph it, you see the limit exists. How can you get it algebraically?

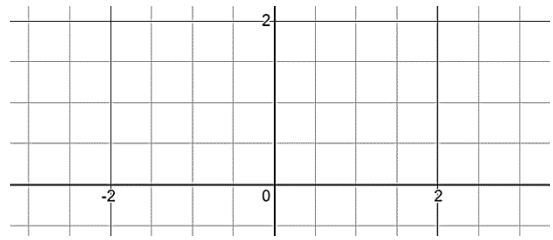


11.2 Using Technology and One-Sided Limits

Ex: Use a calculator's table: Suppose you invest one whole dollar in an account that pays 100% interest compounded n-times per year. Find the balance for each frequency after one year if we compound infinitely many times.

$$\lim_{n\to\infty} 1.00\left(1+\frac{1}{n}\right)^n$$

Ex: Use a calculator's graph: Consider the limit $\lim_{x\to 0}\frac{\sin(x)}{x}$. What happens with direct substitution?

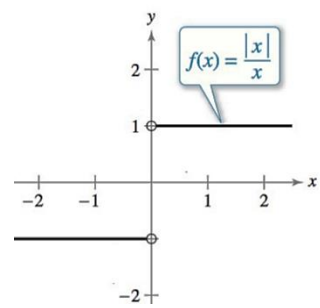


Notation: Instead of saying "as x approaches c from either side" we have notation for that:

$$\lim_{x\to c^-} f(x) = L_1 \text{ or } f(x)\to L_1 \text{ as } x\to c^-$$

$$\lim_{x\to c^+} f(x) = L_2 \text{ or } f(x)\to L_2 \text{ as } x\to c^+$$

Recall the example we did a few lessons ago. Evaluate the following limits.

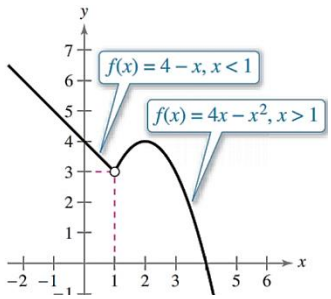
$\lim_{x\to 0^-}\frac{\lvert x\rvert}{x}$	$\lim_{x\to 0^+}\frac{\lvert x\rvert}{x}$	$\lim_{x\to 0}\frac{\lvert x\rvert}{x}$	

Existence of a Limit

If f is a function and c and L are real numbers, then $\lim_{x\to c} f(x) = L$ if and only if both the left and right limits *exist* and are *equal* to L.

Find the limit of $f(x)$ as x approaches 1.

$$f(x) = \begin{cases} 4-x, x<1 \\ 4x-x^2, x>1 \end{cases}$$



$\lim_{x\to 1^-} f(x) =$

$\lim_{x\to 1^+} f(x) =$

$\lim_{x\to 1} f(x) =$

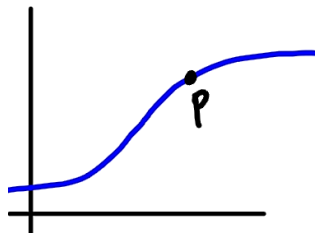
For the function $f(x) = x^2 - 5$, find

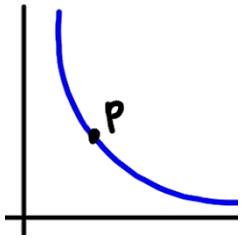
$$\lim_{x\to 0}\frac{f(h+3)-f(3)}{h}$$

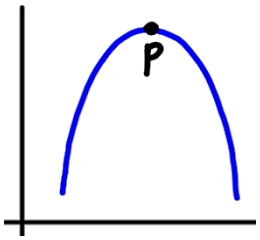
11.3 Day 1 Tangent Line at a Point

Tangent Line: the line that intersects (touches) a graph at a _____ point. If you find the slope of the tangent line, you can find the slope of the graph at a _____ point.

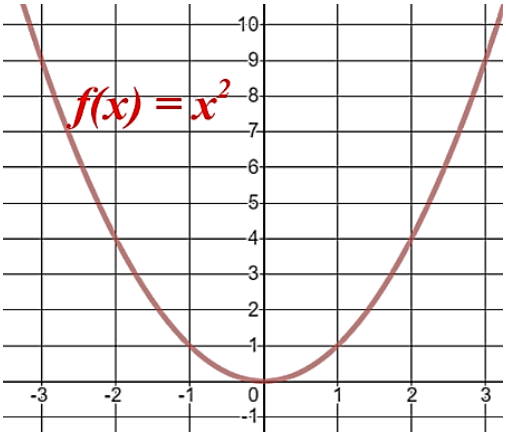
Ex: For the graphs provided, sketch in the tangent line at P.







Ex: For the graph provided, sketch in the tangent line and use it to estimate the slope of the function f(x) at the point (1,1).

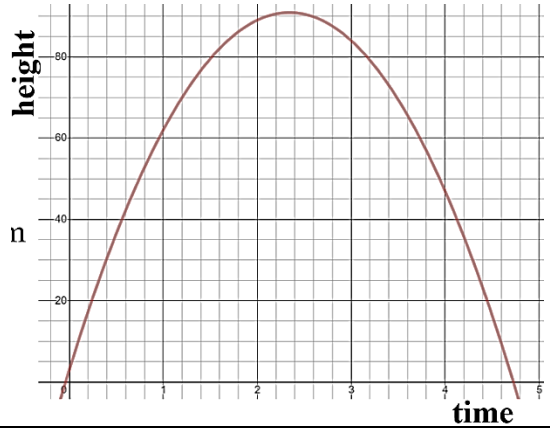


a) The graph for the height of a baseball that was hit into the outfield is given below. Sketch in the tangent line at t = 4.

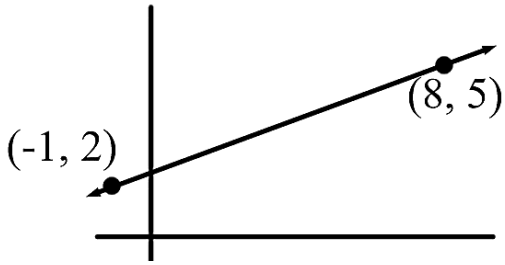
b) Use a program to see what the slope actually is.

c) Explain generally what the slope means in this context.

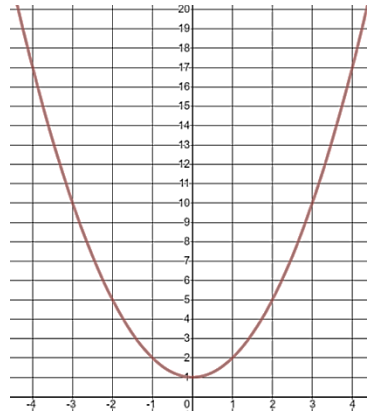
We call distance divided by time the speed, so we would say at 4 seconds the ball's _____ is _____feet per second downward.



Ex: Find the slope of the line between the two points.

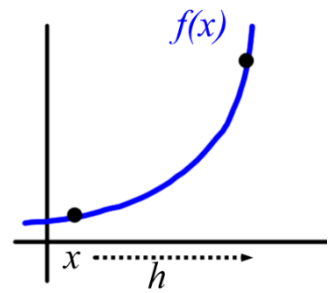


Ex: On the graph $f(x) = x^2 + 1$, find the slope between the points corresponding to x = 1 and the point 3 to right of x = 1.



General Formula for Slope of the Secant Line

$$m_{\text{sec}} = \frac{rise}{run}$$



Definition of the Slope of a Graph

The **slope** m of the graph of f at the point $(x, f(x))$ is equal to the slope of its tangent line at $(x, f(x))$, and is given by

$$m = \lim_{h \to 0} m_{\text{sec}}$$

$$= \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

provided this limit exists.

Ex: Use the limit process and the definition of slope to determine each.

Slope of the graph of $f(x) = x^2 + 1$ at (1,2)	Slope of the graph of $f(x) = x^2 - 2x$ at (-3, 15)

11.3 Day 2 Tangent Line in General (Derivative)

Definition of the Derivative

The **derivative** of f at x is given by

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

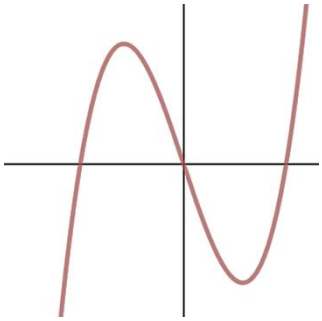
provided this limit exists.

Derivative: function that allows one to determine the slope of the graph of f(x) at _____ point on the graph. You'll sometimes hear it called the "slope equation."

The derivative function is denoted using the following notation: f '(x)

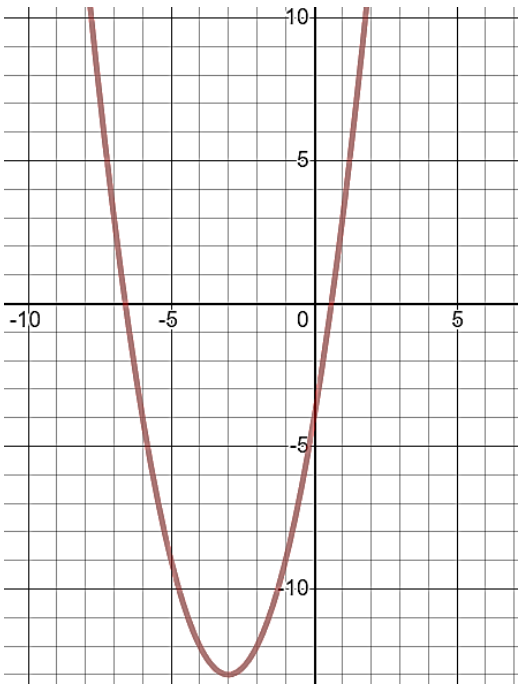
Ex: Find the derivative of $f(x) = x^2 + 1$ and then use it to find the slope of the graph at $x = 1$ and $x = -2$.

Ex: A sketch for the graph of $f(x) = 2x^3 - 6x$ is shown. Find the derivative of f(x) and use it to find the coordinates of the local max and local min.



Ex: **Graphing the Derivative.** a) Find the derivative of $f(x) = x^2 + 6x - 4$

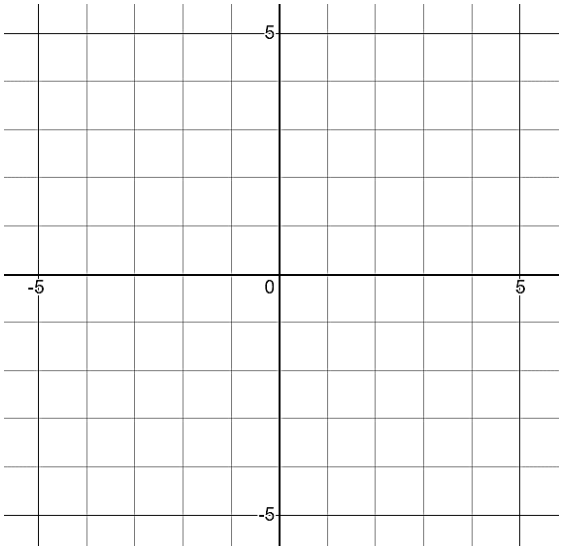
b) You should have found $f'(x) = 2x + 6$. A graph for $f(x) = x^2 + 6x - 4$ is shown. Sketch in the graph for $f'(x)$ on the same graph.

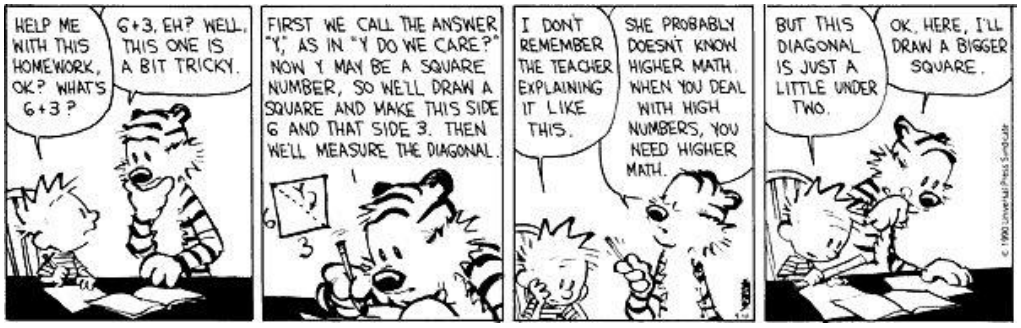


c) Where is the slope of $f(x) = 0$? How is that shown in the graph of $f'(x)$?

d) Where is the slope of $f(x) < 0$? How is that shown in the graph of $f'(x)$?

Ex: In the YOYO, you found the derivative of $f(x) = 2x - 3$ was $f'(x) = 2$. Sketch both on the graph and explain their relationship.





11.3 Day 3 Application of the Derivative

The equation for an object's height above the ground on Earth when moving with an initial upward velocity, v, in feet/second, is $h(t) = -16t^2 + vt + h_0$ where t is the time, in seconds, after it started, and h_0 is the initial height or height at time = 0, measured in feet.

A golf ball is hit off the ground at a speed of 80 ft/sec and leaves at a 30° angle.

a) What is the ball's initial upward speed? Initial horizontal speed?

b) Write the equation for ball's height, h(t).

c) Find the derivative equation.

d) Use the derivative equation to find when the ball reaches max height (when the slope equation/derivative = ___)

e) Use the answer to part (d) to state what the max height is.

f) How long was the ball in the air?

g) Use the answer to (f) and the initial horizontal speed from (a) to determine how far the ball went horizontally.