Ancient Greek Methods of Measuring Astronomical Sizes

Adam Clinch

Contents

Abstract

Mankind's greatest goal has always been to explain why. Why does an acorn fall to the ground? Why does wood burn, but not water? Why can't animals talk and why does the wind blow? Questions like these have fueled civilizations for years, but it's hard to argue that the original spark was not, "Why are we here?" Before a step can be made towards answering that question, people must have felt the need to answer what they meant by "here." At a time when one could look up at the sky and any explanation was equally plausible, the sense of here, the Moon, and the Sun must have held a great sense of mysticism. Although many cultures contributed to the explanations we currently have regarding the relationship between the Earth, the Moon, and the Sun, none established a more mathematical foundation than the Greeks. Before the first year AD, the Greeks had formulated sound methods for measuring the size of the Earth, the distance to the Moon and the Sun, and the size of the Moon and the Sun. The contributions from Greek mathematicians, and their influence on other cultures for centuries to come, will be shared.

A. Clinch (✉)
Capital High School, Helena, MT, USA
e-mail: aclinch@helenaschools.org

B. Sriraman (ed.), *Handbook of the Mathematics of the Arts and Sciences*,
https://doi.org/10.1007/978-3-319-70658-0_66-3

Keywords

Thales · Anaximander · Pythagoras · Aristotle · Aristarchus · Eratosthenes · Posidonius · Archimedes · Hipparchus · Ptolemy · Al Biruni · Copernicus · Geodesy · Cartography · Heliocentric solar system

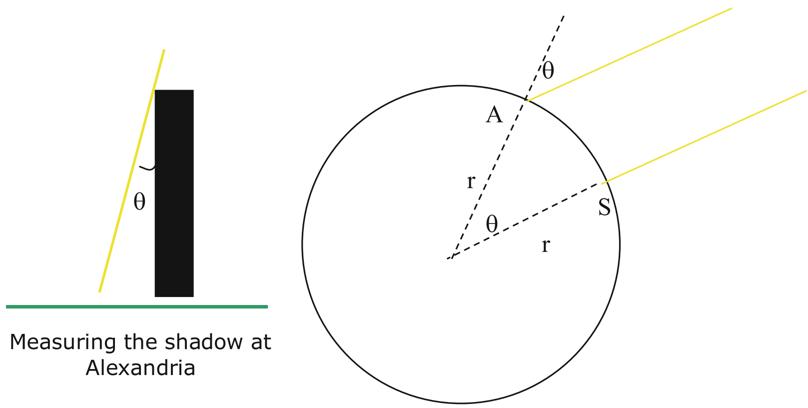
Introduction

As early mathematicians set out to measure the sizes and distances corresponding to the Earth, Moon, and Sun, there was a preface to address. Before calculating any such values, the Greeks had to decide upon the shape of the object they were measuring. If the Earth was a cylinder instead of a sphere, for instance, the method for finding its size would be much different. Though the idea of a cylindrical Earth seems ludicrous now, it was one of the earliest explanations as to the shape of the Earth put forth by the Greek philosopher Thales. Thales lived in the seventh century BC and besides being one of the earliest Greek mathematicians, his contributions include Thales Theorem , an angle subtended by the diameter and another point on a circle is always a right angle, and correctly predicting a solar eclipse. His method for prediction was based off of extensive records, but his ability to date such an astronomical event gained him notoriety. Thales and his predecessor, Anaximander, proposed that the Earth was disk-shaped and floated like a log on a flowing sea. Anaximenes, who studied under Anaximander and lived in the sixth century BC, supposed that the Earth was a rectangle and it, along with the heavens, was held up by compressed air (Brown 1949, 24–25). All three of these scholars failed to supply adequate reasoning for the rising and setting of the Sun and if it went below the disk, or rectangle, when not visible (Heath 1913, 19).

The first correct attempt to describe the Earth as a sphere was made by Pythagoras around 520 BC. Pythagoras is known for founding a school in Crotona, Italy, and for his notorious theorem relating the sides of right triangles. Pythagoras' reasoning for his spherical hypothesis was simply because the sphere was the most perfect shape, a belief not unique to just the Greeks. The first philosopher-mathematician to put any sort of reasoning into this statement was the famous Aristotle in the fourth century BC. He argued that when a lunar eclipse occurs the aberration was appearing because the Earth was casting its shadow on the Moon, the dark portion of the Moon is curved, so the Earth must be curved as well. He also noted that certain stars never set in some locations whereas they may only appear for a few hours a little farther south, a statement which rests its laurels on a spherical Earth.

The final character in this plot is one that plays a significant role later on, Aristarchus of Samos. Though many high school textbooks credit Nicolaus Copernicus as the first person to propose a heliocentric solar system, it was indeed Aristarchus around 290 BC, a fact verified by Archimedes and even mentioned by Copernicus himself. Once it had become more or less accepted that the Earth was spherical in nature, it did not take much time for discussion of its size to arise (Brown 1949, 25–27).

One of the earliest, and arguably the most famous, attempts at measuring the size of the Earth was made by a man named Eratosthenes, who was born in Cyrene in 276 BC. Eratosthenes dabbled in philosophical and scientific fields while studying in Athens and was even thought to have been the tutor for the current monarch during his time as the head librarian at Alexandria. Although his original work is lost to history, his method was recounted by the astronomer Cleomedes and relies on three accepted facts of the time: one, the distance between Alexandria and Syene was 5000 stades or 500 miles; two, Alexandria and Syene lie on the exact same longitudinal line; and three, Syene was located on the Tropic of Cancer. Because Syene was located on the Tropic of Cancer, at noon on the summer solstice, the Sun was directly overhead and cast no shadows, enforced by the famous fact that sunlight would hit the bottom of the local wells. Of course, this was not true in Alexandria and by placing a vertical stick at the city center at the same time, it was found to cast a 7.2° shadow (1/50th of a full circle using a gnomon). Because the Sun is so far away, it is considered to be a point source and all of its rays come in parallel. Observe the exaggerated diagram:

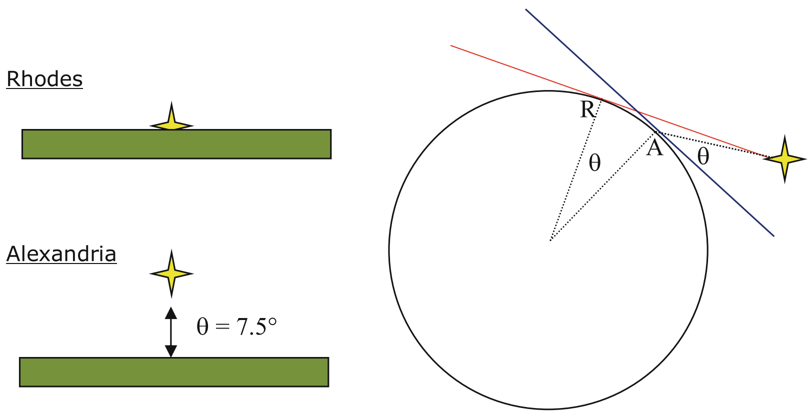


Since θ was measured to be 7.2°, the distance between Alexandria and Syene would therefore be 7.2/360 of the total circumference of the Earth. Setting up a basic proportion, the circumference can be found.

$$\frac{7.2^{\circ}}{360^{\circ}} = \frac{500\text{ miles}}{C_{\text{Earth}}} \rightarrow C_{\text{Earth}} = 500\text{ miles}\left(\frac{360^{\circ}}{7.2^{\circ}}\right) = 25,000\text{ miles}$$

Although his method is completely valid, there are a few inaccuracies in the accepted facts that Eratosthenes used in his measurement. First, Syene sits 37 miles, or 0.53°, north of the Tropic of Cancer. Second, the true distance between Syene and Alexandria is no more than 453 miles. However, it is difficult to hold him accountable for this based on the lack of resources available. Third, Syene and Alexandria differ in longitude by 3.05°. Fourth, the true angle that should have been measured at Alexandria is 7.08°. Lastly, there is debate regarding which stade or stadia Eratosthenes took as his unit. The most accepted belief is that he used the Egyptian stade as was previously used in the calculations. Somewhat miraculously considering all the errors from estimations used in the calculations, Eratosthenes' measurement comes out with an error of only 0.398%, as the accepted value for the circumference of the Earth is 24,901 miles. Interestingly enough though, Eratosthenes must have realized there was error in his method because he raised the value to 25,200 miles unexplainably. Current belief is that he thought the Earth's size would divide evenly into 360 parts and for this case it would mean there are 70 miles per degree. Although Eratosthenes' measurement was incredibly precise compared to the accepted value of the Earth's circumference, there was no way of verifying his number and it was not even taken to be the accepted value for the times. That honor went to Posidonius (Brown 1949, 29–31).

Posidonius lived from 130 to 51 BC and was the head of a school on the Greek island Rhodes where subjects such as philosophy, astronomy, geography, mathematics, and meteorology were taught. In his time, he wrote 52 books, but his claim to fame was in his measurement of the Earth's size and the length of an arc. His calculations would be standards for a few centuries and the benchmark for the earliest maps drawn by Claudius Ptolemy in the late second century AD. Posidonius' method was conceptually similar to Eratosthenes', but the geometry is slightly different. His technique relied on the fact that the star Canopus rose just barely above the horizon for a moment before setting in Rhodes, but on the same night in Alexandria it reached a height of $7°30'$ on the same meridian. The geometry would have looked something like the figures shown.



This is simply another process of measuring the difference in latitude between two places. The rest follows the calculations from Eratosthenes, if you use the fact that the distance between Alexandria and Rhodes was determined to be 500 miles as estimated by mariners. The circumference of the Earth would then be $\left(\frac{360^{\circ}}{7.5^{\circ}}\right) \times$ 500 miles $= 24,000$ miles.

Once again, a sound method was full of errors, which incredibly found its way closer to the true value. The actual angular separation between Rhodes and Alexandria is 5.25°. The reason for the 7.5° measurement is most likely due to refraction and the bending of light. Since Canopus sat near the horizon, its light would be refracted inward by the denser atmosphere, making an object that sits below the horizon appear above. Also, the shear mass of the Earth can bend light, making an object below the horizon again appear above. These two combinations would mean that Canopus did not actually sit on the horizon at Rhodes and all of these together would shrink the value of θ. Posidonius cannot be held responsible for these forms of error as the physics behind these were not understood until René Descartes studied refraction (Snell's Law) in the seventeenth century and Albert Einstein proposed general relativity in the early twentieth century. The other source of error was the distance between the two cities. It is well known that mariners were not terribly precise, as can be seen by the fact that the referenced value fluctuated between 375 miles and 500 miles depending on your source. Strabo, the historian of the time and a supporter of Eratosthenes, used Eratosthenes' estimate of 375 miles when doing the official calculations. The value for the circumference of the Earth would then go down to 18,000 miles, implying 1° in latitude was 50 miles, a vast and a far less precise measure when compared to Eratosthenes'. The reason Posidonius' original guess was closer is because he compensated the overestimate in distances between the cities (actually closer to 360 miles) with another overestimate in their latitude difference. Eratosthenes and Posidonius deserve credit for their measurements, but neither was on the level of Aristarchus of Samos in his attempts to lay a ruler outside the limits of Earth (Brown 1949, 30–32).

Aristarchus of Samos or the "mathematician" lived between 310 and 230 BC. He was well studied in the subjects of music, astronomy, and other areas of science and mathematics, specifically geometry (Heath 1913, 299–300). He is credited with the calculation of a Great Year, the time for the Sun, Moon, and planets (five known at the time) to reset their positions, to be 2434 years (Heath 1913, 314). It is also stated that he correctly calculated when the summer solstice would occur in 281 BC, developed an early version of a Sundial, and wrote a book, which has not survived, detailing his hypothesis of a heliocentric solar system. His greatest accomplishment, however, is his treatise *On the Sizes and Distances of the Sun and Moon*.

The current English translation was conducted by Thomas Heath in 1913 and was based on a manuscript written in the 900 s (Batten 1980, 29). The paper starts with six hypotheses.

1. The Moon is lit by the Sun.
2. The Moon revolves around the Sun.
3. When the Moon is halved, the observer's eye, the Moon and the Sun all lie in one plane.
4. When the Moon is halved, the angle formed by the Moon, observer, and Sun is 87°.
5. The Earth's shadow is twice as large as the diameter of the Moon.
6. The Moon and Sun's diameter is equal to 2° (Heath 1913, 329, 352).

Hypothesis 4 is assumed to have been measured directly, 5 was found by measuring the time it took for a lunar eclipse to occur, and 6 was discovered by measuring how long it took for the Sun to rise and taking that fraction as a fraction of the day (360°). The diagrams drawn up in the paper are provided below.

Because the values in the hypotheses directly affect the numbers obtained by Aristarchus, it is important to understand how he arrived at the stated values. In hypothesis 6, he states that the Moon and Sun's diameters are both equal to 2°. He calculated 2° by measuring the time it took for the Sun to rise or set and taking that fraction as the fraction of the day. For example, say he started measuring the instant he saw the Sun touch the horizon in the evening and stopped 8 min later when the last visible speck of the Sun disappeared. In the 1440 min of a day the Sun traverses 360° through the sky, thus the Sun's diameter is $\left(\frac{8 \text{ min}}{1440 \text{ min}}\right) \times 360^{\circ} = 2^{\circ}$. He also states that the Moon and Sun have equal diameters. He makes this claim based on the observation that the Moon completely, and nearly perfectly, eclipses the Sun.

Hypothesis 5 would have been derived in a similar fashion to hypothesis 4, except this measurement was concerned with how long it took the Earth to completely shade the Moon during a lunar eclipse. He found the Earth's shadow to be twice as large as the Moon, although the correct value is closer to 2.65. The reason for the error is that the measurement depends on the Earth's atmosphere at the time of the eclipse, which also depends on where the Earth and the moon were located in their individual orbits. Because Aristarchus did not realize the variability, he must have determined this value from a single incident.

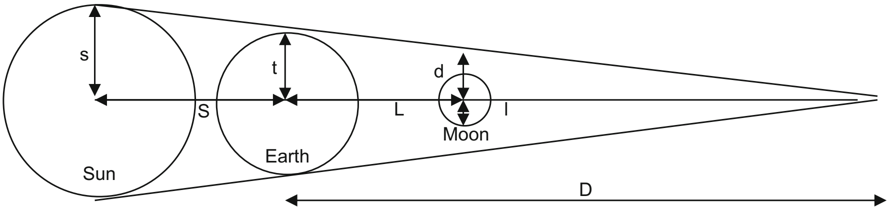
The most crucial and interesting hypothesis is hypothesis 4, which involved measuring the angle made by the Moon, Earth, and the Sun (the angle that would be MES in Fig. 2) when the Moon was exactly half illuminated. Although Fig. 2 is obviously not to scale, in actuality this angle is nearly 90° based on the immense distance between the Earth and the Sun. One can then understand the great difficulty that would accompany this measurement. Again, the value was found by putting an angle in terms of time, so Aristarchus needed to be able to measure exactly when the moon was at half-moon. He then found the angle to be 87°, which will later be shown to have a significant amount of error attached to it. He combined these hypotheses with the diagrams below to arrive at some monumental calculations (Batten 1980, 30–32).

Using the relationships from Fig. 1:

$$\frac{D}{S} = \frac{t}{s - t} \tag{1}$$

$$\frac{d}{t} = \frac{D - L}{D} \rightarrow \frac{D}{L} = \frac{t}{t - d} \tag{2}$$

Multiplying Eqs. 1 and 2 and using the fact that $l/s = L/S$ you get:



t – Radius of the Earth

l – Radius of the Moon

s – Radius of the Sun

L – Distance from the Earth to the Moon

S – Distance from the Earth to the Sun

Fig. 1 The relationship between the Sun, Earth, and Moon

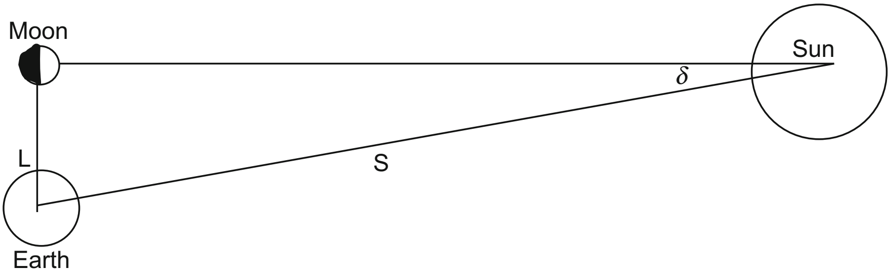


Fig. 2 The relationship when the Moon is half full

$$\frac{D}{S} \times \frac{L}{D} = \frac{L}{S} = \left(\frac{t}{s-t}\right)\left(\frac{t-d}{t}\right) = \frac{t-d}{s-t} \rightarrow \frac{l}{s} = \frac{t-d}{s-t}$$

Manipulating this you get:

$$\frac{l}{s} = \frac{t-d}{s-t} \rightarrow \frac{s-t}{s} = \frac{t-d}{l} \rightarrow \frac{t}{l} + \frac{t}{s} = 1 + \frac{d}{l}$$

You can solve this now for t/l and s/t:

$$\frac{t}{l} + \frac{t}{s} = 1 + \frac{d}{l} \rightarrow \frac{t}{l}\left(1 + \frac{l}{s}\right) = 1 + \frac{d}{l} \rightarrow \frac{l}{t} = \frac{1+\frac{l}{s}}{1+\frac{d}{l}} \rightarrow \frac{t}{l}$$

$$= \frac{1+\frac{l}{s}}{1+\frac{d}{l}} \text{ and likewise } \frac{s}{t} = \frac{1+\frac{l}{s}}{1+\frac{d}{l}}$$

Using hypothesis 5, we let $\frac{d}{l} = 2$. Then using Fig. 2, we let $x = \frac{s}{l} = \frac{1}{\sin(\delta)} = 19$, which is explained later, you arrive at the following values.

$$S/L = 19, s/t = 20/3 = 6.67, t/l = 2.85$$

The value for π had not been calculated yet as it wasn't until Archimedes that π was found to be ≈ 3.14. When π had been determined, we would calculate $\frac{L}{t} = \left(\frac{l}{t}\right)\left(\frac{180}{\theta\pi}\right)$ and $\frac{S}{t} = \left(\frac{s}{t}\right)\left(\frac{180}{\theta\pi}\right)$ where θ is the angular diameter of the Moon which is 2° by hypothesis 6. Using Aristarchus' values, you get $L/t = 10$ and $S/t = 191$.

In Aristarchus' time, trigonometric functions had not yet been invented, so he had to find upper and lower limits for his approximations in a very Euclidean manner. His proofs are extremely in-depth and involve a significant amount of geometric knowledge as well as patience. I will provide a very basic example of how he arrived at his estimation of $L/S = \sin 3° \approx 1/19$.

For an angle θ less than $\pi/2$ you know:

the value of $\sin(\theta/\theta)$ decreases as θ increases
the value of $\tan(\theta/\theta)$ increases as θ increases

It is also known that:

$\sin(\pi/2) = \tan(\pi/4) = 1$
$\sin(\pi/6) = 1/2$
$\tan(\pi/8) = 1/((7/5) + 1)$ where 7/5 is the approximate value for the square root of 2

Using these facts, you can conclude:

For m > 1, $\sin(\pi/2\ \mathrm{m}) > 1/\mathrm{m}$
For m > 2, $\sin(\pi/2\ \mathrm{m}) < 2/\mathrm{m}$
For m > 3, $\sin(\pi/2\ \mathrm{m}) > 3/2\ \mathrm{m}$
For m > 4, $\sin(\pi/2\ \mathrm{m}) < 5/3\ \mathrm{m}$

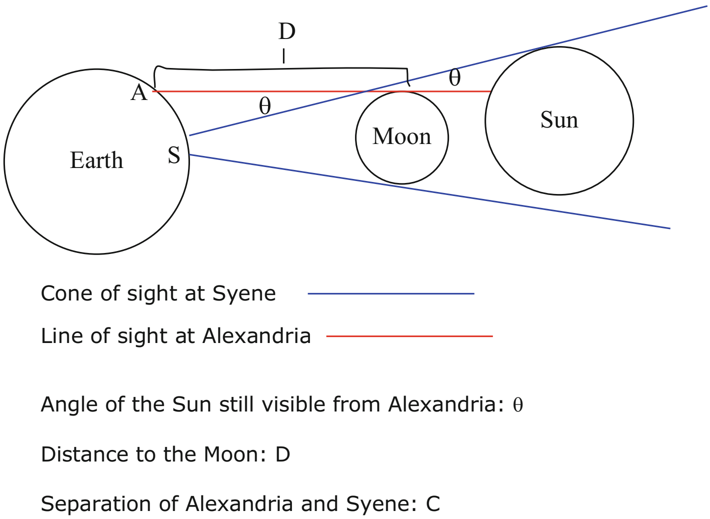
Combining the case for m > 3 and m > 4 you can find a lower and upper limit for $\sin(\pi/2\ \mathrm{m})$.

For our case of sin(3°) you want to find the limits with m = 30, so
5/3 m > $\sin(\pi/2\ \mathrm{m})$ > 3/2 m so 1/18 > sin(3°) > 1/20 (Heath 1913, 333–334).

If you compare Aristarchus' measurements to accepted values you see that the precision is quite poor: $S/L = 389$, $s/t = 109$, $t/l = 3.67$, $L/t = 60.3$, and $S/t = 23{,}455$. The reason for this is twofold. The first is that the actual angular diameter of the Moon and Sun is 1/2°. This is quite astonishing because it is well documented that Aristarchus was one of the first to correctly obtain this value, although it is clearly stated in his piece that the diameter used is 2°. This is one of the reasons that scholars believe that this work was conducted early in his career before he had corrected the value. The second fault is that the angle during the half Moon is actually around

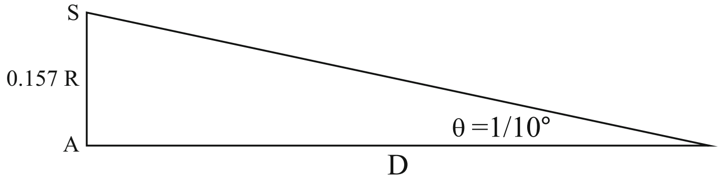
89.86°. Because Aristarchus measured 87°, it is believed that he was off by 6 h when measuring the exact moment the moon was half full. As was stated earlier, this is a very intricate task and with just the naked eye 6 h is nearly the inherent error. The problem is when one is dealing with a $\sec(\theta)$ as θ goes to 90°, slight fluctuations increase the value substantially. For example, the difference between $\sec(\theta)$ when θ goes from 89° to 89.86° is a factor of about 8, so it is easy to see the impact this would cause. Nevertheless, Aristarchus was able to conclude that the Earth was larger than the Moon and the Sun substantially larger than the Earth. Many suppose these findings motivated him to place the Sun at the center, for he believed it to be incorrect that such a large object would be orbiting a smaller one (Batten 1980, 32). Exactly how large these values were varied over time and in the case of the distance to the moon, Hipparchus in the second century BC conducted a more thorough measurement.

Hipparchus' lived from 190 BC to 120 BC and produced a method whose foundation was similar to Eratosthenes' from 100 years earlier, but he relied on the records of a solar eclipse that occurred in his birth year ironically enough. During this particular solar eclipse, it was noted that at Syene the Moon completely eclipsed the Sun, whereas in Alexandria one was still able to see 1/5th of the Sun. This picture is shown below.



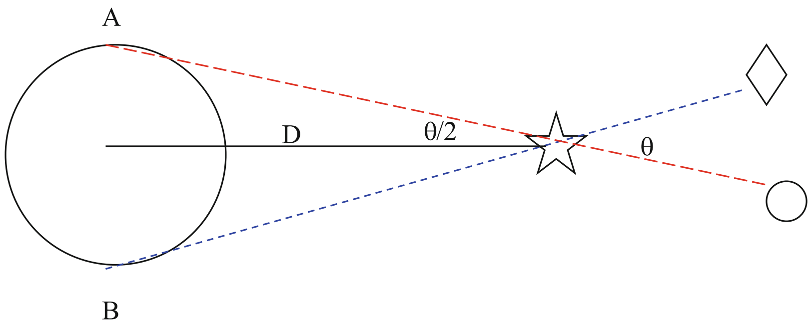
Hipparchus had available the correction that Aristarchus had made about the diameter of the Moon and the Sun, which was $\frac{1}{2}^{\circ}$. Thus, θ would be equal to $\left(\frac{1}{5}\right)\left(\frac{1}{2}^{\circ}\right)$ or $\frac{1}{10}^{\circ}$. At this time, it was reported that Alexandria and Syene were separated by approximately 9°, so we can conclude that $C = \left(\frac{9^{\circ}}{360^{\circ}}\right) \times 2\pi R_{\text{Earth}} \approx$

$0.157 R_{\text{Earth}}$. Since Hipparchus was interested in an estimate, the following diagram is a fair assumption.



Similar to what Aristarchus knew for small angles and considering the fact that Hipparchus was the father of trigonometry having developed chord tables, he could apply the small-angle approximation. Since $\frac{1}{10}^{\circ} = 0.00175$ radians, $D \approx \frac{0.157 R_{\text{Earth}}}{0.00175} = 89.9 R_{\text{Earth}}$. Recalling that the true value is 60.3 R_{Earth} and Aristarchus estimated the distance to the Moon to be 10, the value of Hipparchus' work is clear. There is obvious error in this quick calculation mostly due to the estimation of 1/5 of the Sun being visible at Alexandria as well as the assumptions about the separation and placements of the two sites involved. However, in Ptolemy's *Algamest*, which contains parts of Hipparchus' original work *On Sizes and Distances* [of the Sun and Moon], Hipparchus uses estimates about the Moon and the Sun's parallax as well as two assumptions: the Sun and Moon have the same diameter and the shadow of the Earth during a lunar eclipse is 2.5 times as large as the diameter of the moon, to calculate the mean lunar distance to be 67.3 R_{Earth} and a mean solar distance of 490 R_{Earth}, both of which were dramatic improvements to all prior measurements (Toomer 1974, 126–140). After Hipparchus had laid the foundation for using parallax, Claudius Ptolemy continued with this to again measure the distance to the moon.

Claudius Ptolemy lived from 90 AD to 168 AD and besides being a well-known mathematician and astronomer, he was most famous for his mapmaking skills. Before discussing Ptolemy's work, the concept of parallax will be discussed. Imagine two people (A and B) on either side of the Earth observing a star.



Now it should be clear that since θ can be directly measured, $D = \frac{R_{\mathrm{Earth}}}{\tan\left(\frac{\theta}{2}\right)}$.

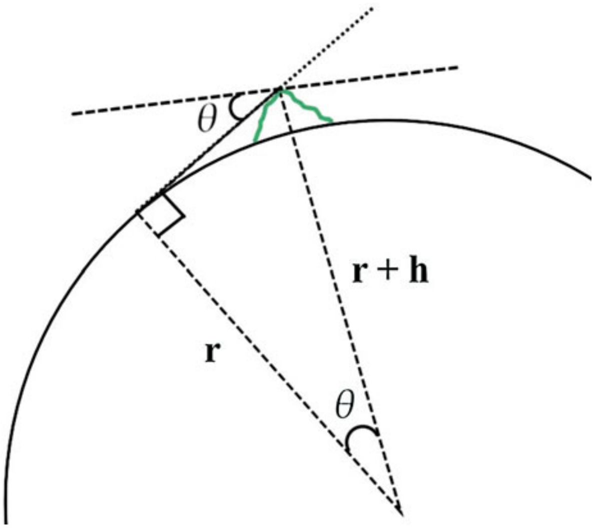
What Ptolemy did was directly measure the parallax of the moon between two cities whose distance was known. The diagram is the same as the one above, so by having three measurements of the location of the moon taken (one at A, one at B, and one halfway between) the distance to the moon can be extracted. With this method Ptolemy found the distance to the Moon to be 59 R_{Earth} (Tassoul and Tassoul 2004, 31). Although his value didn't improve substantially on the precision of Hipparchus' measurement, the method of parallax set forth is still used to calculate the distance to stars whose parallax is observable. This method was to be used in 1769 to calculate the solar parallax and thus the distance from the Earth to the Sun by using Venus's projection as it traveled across the face of the Sun. By measuring the longitudinal difference of the projection on the Sun at separate places on the Earth, Thomas Hornsby calculated a solar parallax of $8.78''$ and Leonhard Euler measured $8.82''$. The accepted value is $8.79''$, which shows the usefulness of the 1600-year-old method (Metz 2007, 13).

By 168 AD the Greeks had measured the size of and distances between the Earth, Moon, and Sun to astounding accuracy. It would be centuries before other civilizations in India, the Roman empire, and the Islamic world would develop techniques of their own. To demonstrate an example, the work of Abū Rayḥān Muḥammad ibn Aḥmad Al-Bīrūnī, going forward referred to as Al-Biruni, will be presented, as the geometric and trigonometric derivation closely resembles that of previous Greek work.

Al-Biruni was born in 973 AD in modern day Uzbekistan. As a physicist, mathematician, and astronomer, he contributed greatly to many fields, specifically the trigonometric functions for and values corresponding to tangent, cotangent, secant, and cosecant. Using these and the constructed trigonometric tables, Al-Biruni was able to measure the size of the earth (Katz 2009).

Of significant importance to those of Islamic faith is the distance and direction from any point on Earth to Mecca. With this in mind, in the ninth century Sultan al-Mamun ordered two surveying teams to measure the distance between two points whose latitude differed by 1°. Trekking through the desert, the teams found that 1° of separation corresponded to between 56 and 57 Arabic miles or 67 modern-day miles (Biruni and Sezgin 1967). Wishing to replicate the experiment, but unwilling to find funds, Al-Biruni developed a far less strenuous method.

His technique relied on the diagram below. By setting $r =$ radius of the Earth, $h =$ height of the mountain, and $\theta =$ the angle of declination, we can use the law of sines to derive the following.



$$\frac{r}{\sin(90-\theta)} = \frac{r+h}{\sin(90)}$$

$$\frac{r}{\cos(\theta)} = \frac{r+h}{1}$$

$$r = r\cos(\theta) + h\cos(\theta)$$

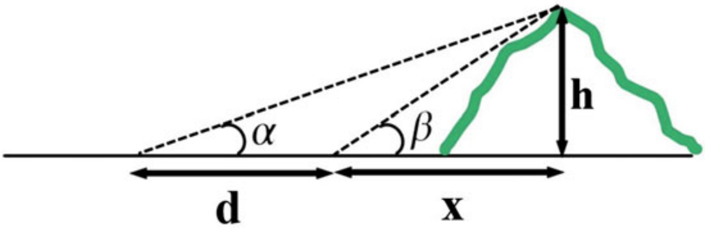
$$r - r\cos(\theta) = h\cos(\theta)$$

$$r(1-\cos(\theta)) = h\cos(\theta)$$

$$r = \frac{h\cos(\theta)}{1-\cos(\theta)}$$

In order to determine the radius of the Earth, Al-Biruni had reduced the question down to finding the height of a given mountain that has the capacity to observe the horizon, thus creating the 90° angle via the tangent line of sight, and the angle of declination from that line of sight.

Using a mountain in modern-day Pakistan, Al-Biruni used a well-known surveyor's technique to first determine the height of the mountain (Mercier 1994, 183). Reference the diagram below.



By taking measurements for the angle of inclination at two points, α and β, and measuring the distance between those points, d, he could determine the height of the mountain through the following trigonometric setups and algebraic manipulations.

$$\text{Note : } \tan(\alpha) = \frac{h}{d+x} \text{ and } \tan(\beta) = \frac{h}{x}$$

Solving both for x, we obtain the following.

$$x = \frac{h - d\tan(\alpha)}{\tan(\alpha)} \text{ and } x = \frac{h}{\tan(\beta)}$$

Setting the equations equal to one another, we can solve for h.

$$h\tan(\beta) - d\tan(\alpha)\tan(\beta) = h\tan(\alpha)$$

$$h(\tan(\beta) - \tan(\alpha)) = d\tan(\alpha)\tan(\beta)$$

$$h = \frac{d\tan(\alpha)\tan(\beta)}{\tan(\beta) - \tan(\alpha)}$$

Using an astrolabe to measure these angles and pacing off the distance d, the values of these measurements are not documented. What is recorded is that upon measuring them, he determined the height of the mountain, h, was 652 cubits (about 0.32 km) and the angle of declination, θ, was $0°34'$ or $0.57°$ (Biruni 1956). Substituting these values into the formula for the radius of the Earth, we obtain $r = 6466$ km. Al-Biruni states the value at 6336 km due to an incorrect conversion when recording the angle in the Babylonian sexagesimal system. He also, like others before him, did not have the knowledge or tools to account for the refraction of light when conducting his measurements. Fortunately for Al-Biruni, the errors were favorable, as the radius of the earth is 6371 km, producing an error of just over 0.5%.

Conclusion

To rate the importance of the contributions made by Eratosthenes, Posidonius, Hipparchus, Ptolemy, Aristarchus, and later non-Greek mathematicians such as Al-Biruni would be an impossible task. They gave precise values that were to be used in cartography, sea travels, calendars, and astronomy for a 1000 years. To be able to say that the Earth was a sphere, the Moon was "x" times smaller, and the Sun a distance "y" times farther than the Moon was a substantial achievement for men who had nothing but basic geometry and versions of a gnomon. On a more philosophical level, they gave a quantitative and qualitative measurement of this home we call Earth located in the neighborhood of our solar system. They built the foundation that astronomers such as Kepler, Galileo, Copernicus, and Halley would later stand on, 1500 years later. They were the first pioneers of the last frontier, in our great quest to answer, "Why?"

Cross-References

▶ Classical Greek and Roman Architecture: Examples and Typologies
▶ Classical Greek and Roman Architecture: Mathematical Theories and Concepts
▶ Egyptian Architecture and Mathematics

References

Batten AH (1980) Aristarchos of Samos. Royal Astronomical Society of Canada, 75. Retrieved from http://articles.adsabs.harvard.edu//full/1981JRASC7529B/0000029.000.html

Biruni (1956) Al-Qanun al-Masudi [Masudic Canon], or An encyclopedia of astronomical sciences, 3 vols. Osmani Oriental Publications Bureau, Hyderabad

Biruni MAA, Sezgin F (1967) The determination of the coordinates of positions for the correction of distances between cities =: Taḥdīd nihāyāt al-amākin li-taṣḥīḥ masāfāt al-masākin. Institute for the History of Arabic-Islamic Science at the Johann Wolfgang Goethe University, Frankfurt am Main

Brown LA (1949) The story of maps. Little, Brown and Company, Boston

Heath T (1913) Aristarchus of Samos the ancient Copernicus. Clarendon Press, Oxford

Katz VJ (2009) A history of mathematics: an introduction. Addison-Wesley, Boston

Mercier RP (1994) Geodesy. In: Harley JB, Woodward D (eds) The history of cartography, book 1, vol 2. University of Chicago Press, Chicago/London

Metz D (2007) William Wales and the 1769 transit of Venus: puzzle solving and the determination of the astronomical unit. Sci Educ. https://doi.org/10.1007/s11191-006-9073-3

Tassoul JL, Tassoul M (2004) A concise history of solar and stellar physics. Princeton Press, Princeton

Toomer GJ (1974) Hipparchus on the distances of the sun and moon. Arch Hist Exact Sci 14. https://doi.org/10.1007/BF00329826