Application of Binomial Probabilities: Airline Overbooking

Real-World Application from Chapter 4

Airlines routinely overbook flights because of two reasons: 1) not all people show up for the flight and 2) they can make more money if they overbook and some don’t show up. This is risky business however, because if they overbook and more people show up than they have seats for, they will have to pay to compensate people for overbooking.

Scenario: Let’s say a flight has 120 seats each selling for $400. If the airline overbooks, they will pay willing customers the price of their ticket back plus a 20% overbooking penalty to give up their seat until the plane is not overbooked.

1. What would a customer be compensated if they are willing to give up their seat in the case of overbooking?

2. Let’s say the flight sold 120 seats. How much money did the airline make on the seats?

3. Let’s say the flight only sold 115 seats. How much money did the airline make on the seats?

4. Let’s say the flight was overbooked with 125 people in total and all 125 people show up. How much money did the airline make in total after paying the overbooking compensation to 5 willing customers?

5. Let’s say the flight was overbooked with 125 people in total but only 118 showed up. How much money did the airline make on the seats?

6. Why was the answer to #5 different from #4? Explain in general when it is advantageous for the airline to overbook.

7. Overbooking the plane to 125 people in total (with 125 people showing up) will earn the airline just as much money in total as only booking it to how many below 120 (so underbooking it to what)? Explain clearly why it is advantageous for the airline to overbook based on your answer.
**Scenario**: Because not all airline passengers show up for their reserved seat, the airline sells 125 tickets for a flight that holds 120 passengers. The probability that each individual passenger shows up is 0.90, and assume the passengers behave independently.

1. Why can you use the binomial formula for this situation (that is, what makes it a binomial experiment)? *Look in book or notes if you need help.
   i. 
   ii. 
   iii. 
   iv. 

2. State the following for this binomial experiment: 
   \[ n = \_\_\_\_\_\_\_ \quad p = \_\_\_\_\_\_\_ \quad q = \_\_\_\_\_\_\_ \]

3. What is the probability that exactly 120 people show up?

4. What is the probability that every passenger who shows up can take the flight?

5. What is the probability that the flight departs with at least one empty seat?

6. What is the probability the airline will have to pay at least one person for overbooking?

7. Calculate the mean and standard deviation for this binomial distribution based on the values you gave in Question 2 above, where the mean is the average number of customers they expect to show up.

8. Using the values from Question 7 and the Empirical Rule, the airline expects that 68% of flights like this will have between what two numbers of customers showing up?

9. What is the z-score for the situation where 121 customers show up (that is, how many standard deviations away is this)? Is this unusual?

10. Let’s say the airline will overbook until there is at least a 5% chance that they will have to pay at least one person for overbooking. By guessing and checking, change the value of “n” until you find how many seats they will sell in total until this occurs. That is, how much can they overbook by until this happens?