MATHEMATICS • PRACTICE TEST 2 • EXPLANATORY ANSWERS

Question 1. The correct answer is C. The correct approach for this problem is to first distribute a and then combine the two a-terms. Doing so, we get $a(4 - a) - 5(a + 7) = 4a - a^2 - 5a - 35 = -a^2 - a - 35$. If you chose **A**, you may have thought that $a \cdot a = a$ and did not get the a^2-term when you distributed. If you chose **B**, you may have made the same error as above but also forgotten to distribute -5 over *both terms* of $(a + 7)$. If you chose **D**, you may have forgotten to distribute -5 over *both terms* of $(a + 7)$. If you chose **E**, you may have incorrectly combined like terms by thinking $4a - a^2 - 5a = (4 - 1 - 5)a^3$. Remember, you can only combine terms with identical variable parts.

Question 2. The correct answer is G. You can order the numbers by considering the decimal equivalent of $\frac{1}{4} = 0.25$ and writing each fraction in an equivalent form with 100 as the denominator: $0.2 = \frac{20}{100}$, $0.03 = \frac{3}{100}$, and $\frac{1}{4} = \frac{25}{100}$. The fractions can then be ordered based on the magnitude of the numerator: $\frac{3}{100} < \frac{20}{100} < \frac{25}{100}$ or $0.03 < 0.2 < \frac{1}{4}$. If you chose **F**, you may have incorrectly used $2 < 3 < 4$. If you chose **H** or **J**, you may not have considered that $\frac{1}{4} = 0.25$. If you chose **K**, you may have incorrectly given the order from *greatest* to *least*.

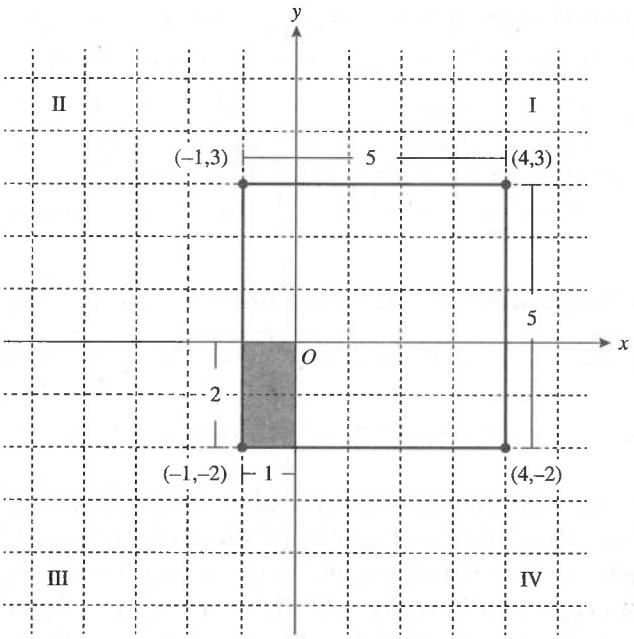
Question 3. The correct answer is C. Because $x^2 + 4 = 29$, you can solve the equation for x^2, $x^2 + 4 = 29 \Leftrightarrow x^2 = 25$. Substitute the value of x^2 into the expression $x^2 - 4$ to calculate the value the expression is equal to $x^2 - 4 = 25 - 4 = 21$.

If you chose **A**, you might have solved $x^2 + 4 = 29$ for the principal square root of x, $x^2 + 4 = 29 \Leftrightarrow x^2 = 25 \Leftrightarrow \sqrt{x^2} = \sqrt{25} \Leftrightarrow x = 5$ and neglected to evaluate the expression $x^2 - 4$. If you chose **B**, you might have incorrectly combined $x^2 + 4 = 29$ with $x^2 - 4$ and solved for x, $x^2 + 4 = 29 - 4 \Leftrightarrow x^2 = 29 - 4 - 4 \Leftrightarrow x^2 = 21 \Leftrightarrow x = \sqrt{21}$. If you chose **D**, you might have only solved $x^2 + 4 = 29$ for x^2, $x^2 + 4 = 29 \Leftrightarrow x^2 = 25$. If you chose **E**, you might have incorrectly manipulated $x^2 + 4 = 29$ and only solved for x^2, $x^2 + 4 = 29 \Leftrightarrow x^2 = 29 + 4 \Leftrightarrow x^2 = 33$.

Question 4. The correct answer is F. As shown in the following figure, the given vertices form a 5-unit-by-5-unit rectangle (or square), which has an area of $5 \times 5 = 25$ square coordinate units. The portion of that square lying in Quadrant III is shown in the figure as the 2-unit-by-1-unit shaded rectangle, which has an area of $2 \times 1 = 2$ square coordinate units. Therefore, the percent of the total area of the square lying in

Quadrant III $= \frac{\text{the area of the shaded rectangle}}{\text{the area of the square}} \times 100\% = \frac{2}{25} \times 100\% = 8\%$.

MATHEMATICS • PRACTICE TEST 2 • EXPLANATORY ANSWERS



Question 5. The correct answer is B. Assuming the cost increased *linearly* is equivalent to assuming the cost increased *at a constant rate.* Because the cost of the clothing increased from \$620 to \$1,000 in 10 years, the constant rate is equal to $\frac{\$1{,}000-\$620}{10 \text{ years}}$, or \$38 per year.

Therefore, the cost of the family's clothing in 1991 (6 years after 1985) is:

\$620 + (6 years)(\$38 per year) = \$620 + \$228 = \$848.

Question 6. The correct answer is K. You can let x represent the certain number so that $\sqrt{x} = 9.2371$ or $x = 9.2371^2$ after squaring both sides of the equation. Now, $9^2 < x < 9.5^2 \rightarrow 81 < x < 90.25$ and $90.25 < 99$, so x is between 81 and 99. If you chose **F**, you could have incorrectly thought that $x^2 = 9.2371$ and used the fact that $3^2 = 9$ and $4^2 = 16$. If you chose **G**, you could have incorrectly thought that $2x = 9.2371$ or $x = \frac{9.2371}{2}$, which is between $\frac{8}{2} = 4$ and $\frac{10}{2} = 5$. If you chose **H**, you could have incorrectly thought that $x = 9.2371$, which is between 9 and 10. If you chose **J**, you could have incorrectly thought that $\frac{x}{2} = 9.2371$ or $x = 2(9.2371)$, which is between $2(9) = 18$ and $2(9.5) = 19$.

Question 7. The correct answer is E. There are 10 − 2 = 8 pieces of candy that are NOT grape, and $\frac{8}{10} = \frac{4}{5}$. If you chose **A**, you may have incorrectly found the probability that the candy randomly picked IS grape. If you chose **B**, you may have incorrectly found the probability of *grape* out of the four flavors. If you chose **C**, you may have incorrectly thought that the candy randomly picked is either grape or *not grape*, so there is a 50% chance of *not grape*. If you chose **D**, you may have incorrectly found the probability of *not grape* out of the four flavors.

Question 8. The correct answer is K. The coordinates of the midpoint, (x_m,y_m), of a segment with endpoints (x_1,y_1) and (x_2,y_2) are $\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$. Substituting, we get $1 = \frac{x_1+(-3)}{2} \Leftrightarrow 2 = x_1 - 3 \Leftrightarrow x_1 = 5$, and $2 = \frac{y_1+4}{2} \Leftrightarrow 4 = y_1 + 4 \Leftrightarrow y_1 = 0$. Therefore the midpoint is (5,0), **K**. An alternative approach to this problem would be to plot point *B* and the midpoint and then use the concept of displacement to find the coordinates of *A*. Because point *B* is 4 coordinate units *left* and 2 coordinate units *up* from the midpoint, point *A* must be 4 coordinate units *right* and 2 coordinate units *down* from the midpoint, (1,2). So, once again, we find the coordinates of point *A* to be (1 + 4,2 − 2) = (5,0). If you chose **F**, you may have used the correct midpoint formula but interchanged the coordinates of the midpoint and point *B*. If you chose **G**, you may have incorrectly written minus signs in the formula where there should be plus signs and used (1,2) and point *B* as the 2 endpoints. If you chose **H**, you found the midpoint of the segment with endpoints (1,2) and point *B*. If you chose **J**, you may have incorrectly written minus signs in the formula where there should be plus signs.

Question 9. The correct answer is B. Let x represent the number of customers Andrea's company had 1 year ago. Paraphrasing, the 116 customers are 8 more than twice the number of customers the company had 1 year ago. Replacing the "are" with "=" and translating both halves of the sentence into symbols, we get $116 = 2x + 8$. Solving this equation, we get $x = 54$, which is **B**. If you chose **A**, you may have added 8 to x rather than to $2x$, giving you the incorrect equation $116 = 2(x + 8)$. If you chose **C**, you may have subtracted 8 from $2x$ rather than adding it, giving you the incorrect equation $116 = 2x - 8$. If you chose **D**, you may have subtracted 8 from x and then doubled that, obtaining the incorrect equation $116 = 2(x - 8)$. If you chose **E**, you may have written the incorrect equation $116 = x + 2(8)$. Please remember that "8 more than" means to add 8 to something. Add it to what? Add it to "twice the number of customers the company had 1 year ago," which is $2x$.

Question 10. The correct answer is H. You can find the set amount per foot of fence by subtracting the $500.00 fee from the estimate given: $2,200.00 − $500.00 = $1,700 and dividing that result by 200 ft: $\frac{\$1,700}{200\text{ ft}} = \$8.50$ per foot of fence. If you chose **F**, you may have incorrectly subtracted 200 from 2,200 and divided that result by 500. If you chose **G**, you may have incorrectly added 200 to 2,200 and divided that result by 500. If you chose **J**, you may have forgotten to subtract 500 from 2,200 before dividing by 200. If you chose **K**, you may have incorrectly used the sum of 500 and 2,200 divided by 200.

MATHEMATICS • PRACTICE TEST 2 • EXPLANATORY ANSWERS

Question 11. The correct answer is C. You can solve this problem numerically by considering all the possible pairs of integer values representing the width and length of the room for which the area equals 180 ft^2. Then you can find the pair of values for which the perimeter equals 54 ft (that pair of values is 12 ft and 15 ft in the shaded region of the following table).

Width (ft)	1	2	3	4	5	6	9	10	12
Length (ft)	180	90	60	45	36	30	20	18	15
Area (ft^2)	180	180	180	180	180	180	180	180	180
Perimeter (ft)	362	184	126	98	82	72	58	56	54

You can also approach this problem analytically by solving the system of equations $2w + 2l = 54$ and $wl = 180$, which result from using the perimeter and the area formulas, respectively.

Notice that $2w + 2l = 54$ is equivalent to $w + l = 27$, and solving $wl = 180$ for l gives $l = \frac{180}{w}$. Substituting this value for l, you obtain $w + \frac{180}{w} = 27$; multiplying both sides by w and then subtracting $27w$ from both sides leads to $w^2 - 27w + 180 = 0$. Factoring the quadratic gives the equivalent equation $(w - 12)(w - 15) = 0$, with solutions $w = 12$ or $w = 15$; the corresponding solutions for l are $\frac{180}{w} = 15$ and $\frac{180}{w} = 12$.

You may notice that the work involved in factoring is essentially the same work required in the numerical method, so the analytical method is a little less efficient. You may also notice that the system of equations is symmetric in w and l, so that the solutions for *either w or l* give the dimensions of the living room.

Question 12. The correct answer is J. You can consider that \$10.00 − \$4.25 = \$5.75 and $\frac{\$5.75}{\$0.25} = 23$ quarters. If you chose **F**, you may have found the number of quarters in \$25.00 and divided that number by 10. If you chose **G**, you may have incorrectly considered the number of quarters in \$4.25 only. If you chose **H**, you may have incorrectly used the price of 25 candies at Carrie's Chocolate Shop. If you chose **K**, you may have incorrectly divided the number of quarters in \$25 by 10 and subtracted that result from the number of quarters in \$10.00.

Question 13. The correct answer is B. The average price per candy is the total price divided by the number of candies. This is equal to $\frac{\$3.75}{20} = \$0.1875$ or \$0.19 when rounded to the nearest \$0.01. If you chose **A**, you may have incorrectly found the change in price from 5 candies to 20 candies and divided that result by 20. If you chose **C**, you may have incorrectly used the price of 20 candies at Carrie's Chocolate Shop. If you chose **D**, you may have incorrectly multiplied the cost of 5 candies at Carrie's Chocolate Shop by 4 and divided that result by 20. If you chose **E**, you may have incorrectly multiplied the cost of 5 candies at Tamika's Treat Shop by 4 and divided that result by 20.

Question 14. The correct answer is F. You can verify that the relationship is linear by noting that each increase of 5 candies results in an increase of \$1.00 in price so that the slope of the line is $m = \frac{1.00}{5} = 0.20$. The equation can then be written in the form $c = 0.20n + b$. You can solve for the value of b by substituting any one of the (n,c) ordered pairs for Carrie's Chocolate Shop into the equation. Using (5,1.50), we have $b = 1.50 - 0.20(5) = 0.50$. Therefore, the correct equation is $c = 0.20n + 0.50$. If you chose **G**, you may have incorrectly thought that the slope of the line would be $\frac{1.50}{5}$ and $b = 0$. If you chose **H**, you may have incorrectly found the slope to be the change in price at Tamika's Treat Shop relative to the change in price at Carrie's Chocolate Shop and the value of b as the price for 5 candies at Carrie's Chocolate Shop. If you chose **J**, you may have incorrectly thought the \$1.00 increase in price is because of an increase of 1 piece of candy and solved for b using the equation $b = 1.50 - 5(1)$. If you chose **K**, you may have incorrectly found the slope by dividing the difference in price of 30 candies at the 2 shops by the difference in price of 25 candies at the 2 shops and using an ordered pair from Carrie's Chocolate Shop to find the value of b.

Question 15. The correct answer is B. To solve the quadratic equation $x^2 - 36x = 0$ for x, you would factor the left side to apply the zero product rule to $x(x - 36) = 0$. Thus, $x = 0$ or $x - 36 = 0$ implies $x = 0$ or 36. The solution given as an answer choice is 36.

If you chose **C**, you probably divided 36 by 2. If you chose **D**, you probably dropped the x in the second term and solved $x^2 = 36$ for a positive value. If you chose **E**, you probably dropped the x in the second term and solved $x^2 = 36$ for negative value because there was a negative sign in the original equation.

Question 16. The correct answer is J. Using the properties of supplementary angles, we find the measures of $\angle EDF = 180° - 148° = 32°$and $\angle EFD = 180° - 140° = 40°$. Then, by the 180° rule for triangles, the measure of $\angle DEF = 180° - 32° - 40° = 108°$. If you chose **F**, you may have forgotten to subtract the measures of $\angle EDF$ and $\angle EFD$ from 180° (180° rule for triangles). If you chose **G**, you may have incorrectly subtracted 140° from 148° and added that result to 90°. If you chose **H**, you may have incorrectly subtracted 140° rather than 148° from 180° for angle $\angle EDF$. If you chose **K**, you may have incorrectly subtracted 148° rather than 140° from 180° for angle $\angle EFD$.

Question 17. The correct answer is E. Assume the base of the box is 9 inches by 9 inches. Because both dimensions are divisible by 3, 3 rows with 3 notepads in each row placed edge to edge will exactly cover the base. We can call this 1 layer. Because the box is 12 inches high, it will hold exactly 4 such layers. Therefore, 4 layers with 9 notepads in each layer will result in 36 notepads in the box, **E**. Because all 3 dimensions of the box are evenly divisible by the edge length of the cubical notepads, this can also be thought of as $\frac{9}{3} = 3$ notepads wide, $\frac{9}{3} = 3$ notepads long, and $\frac{12}{3} = 4$ notepads high, and $3 \times 3 \times 4 = 36$. If you chose **A**, you may have obtained 3 notepads long by 3 notepads wide by 4 notepads high but then added instead of multiplying. Because there will be no gaps or overlap of the notepads in the box, another method would be to divide the volume of the box by the volume of a single notepad. Using

the formula for the volume of rectangular prism, $V = l \times w \times h$, we get $\frac{\text{Volume of the box}}{\text{Volume of a notepad}} = \frac{9(9)(12)}{3(3)(3)} = 36$. If you chose **B**, you may have been thinking this way but incorrectly used the area of $\frac{1}{2}$ of the faces of the box instead of volume of the box. If you chose **C**, you may have thought the box could hold only one layer 3 notepads long by 4 notepads high, and you did not consider how many notepads it could hold widthwise. If you chose **D**, you may have been trying to divide the volume of the box by the volume of a single notepad, but you incorrectly found the volume of the box to be $2(9 \times 9 + 9 \times 12 + 9 \times 12)$, which is the surface area instead.

Question 18. The correct answer is J. Evaluate the function f for $x = -4$, $f(-4) = -4(-4)^3 - 4(-4)^2 = -4(-64) - 4(16) = 256 - 64 = 192$. If you chose **F**, you might have dropped the negative sign on the first 4 in the function: $4(-4)^3 - 4(-4)^2$. If you chose **G**, you might have dropped the negative signs on both 4s in the function: $4(-4)^3 + 4(-4)^2$. If you chose **H**, you might have dropped the negative on the first 4, evaluated the function at $x = 4$, and incorrectly calculated the exponents in the function: 4(4)3 − 4(4)2. If you chose **K**, you might have dropped the negative sign on the second 4 in the function: $-4(-4)^3 + 4(-4)^2$.

Question 19. The correct answer is A. Solving the first equation for x gives you $x = 4 - 2y$. Plugging in the expression that x equals into the second equation gives you $-2(4 - 2y) + y = 7$. You can solve for y as follows $-8+4y+y=7 \Leftrightarrow -8+5y=7 \Leftrightarrow 5y=15 \Leftrightarrow y=3$ Now, take the value for y and plug it in for y in one of the original equations and solve for x, $x+2(3)=4 \Leftrightarrow x+6=4 \Leftrightarrow x=-2$. Therefore, (−2,3) is the solution for this system of equations.

If you chose **B**, you might have plugged −1 in for x in the first equation and solved for y without checking to see if those values hold for the second equation, $-1+2y=4 \Leftrightarrow 2y=5 \Leftrightarrow y=2.5$. If you chose **C**, you might have plugged 1 in for x in the first equation and solved for y without checking to see if those values hold for the second equation, $1+2y=4 \Leftrightarrow 2y=3 \Leftrightarrow y=1.5$. If you chose **D**, you might have plugged 1 in for y in the first equation and solved for x without checking to see if those values hold for the second equation, $x+2(1)=4 \Leftrightarrow x+2=4 \Leftrightarrow x=2$. If you chose **E**, you might have plugged 0 in for y in the first equation and solved for x without checking to see if those values hold for the second equation, $x+2(0)=4 \Leftrightarrow x+0=4 \Leftrightarrow x=4$.

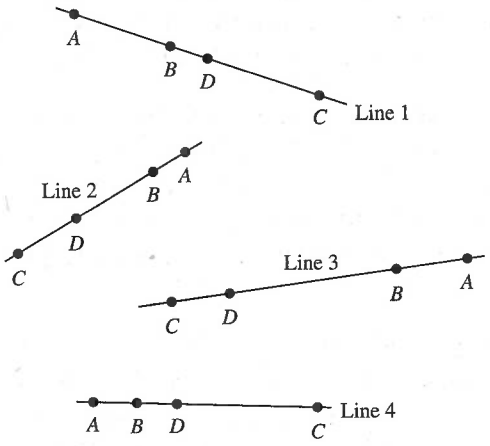
Question 20. The correct answer is G. By definition, $\log_x 36$ is the power of x that it would take to be 36. That power is 2 from the right side of the equation. So, $x^2 = 36$. This equation is satisfied when $x = 6$.

The most common wrong answer is **K**, which is the solution to $x \cdot 2 = 36$.

Question 21. The correct answer is C. The area of a rectangle is given by area = length × width, or $A = lw$. The area of the picture before cutting is $A = 7(5) = 35$ square inches. The area enclosed by the frame is $A = 6(4) = 24$ square inches. Subtracting, we see 35 − 24 = 11 square inches must be cut off the picture to make it exactly

fit into the frame. That is **C**. If you chose **A**, you may have incorrectly thought that because the picture is 1 inch longer and 1 inch wider than the frame, a total of 2 inches must be cut off the picture. Please note that the problem asks for how much *area* must be cut off the picture. If you chose **B**, you may have incorrectly thought that because the picture is 1 inch longer and 1 inch wider than the frame, and the frame is 4 inches by 6 inches, the total area cut off must be 1(6) + 1(4) square inches. If you chose **D**, you may have incorrectly thought that because the picture is 1 inch longer and 1 inch wider than the frame, and the picture is 5 inches by 7 inches, the total area cut off must be 1(5) + 1(7) square inches. If you chose **E**, you may have incorrectly thought that because the picture is 1 inch longer and 1 inch wider than the frame, the amount of area that needs to cut off is 1 × 1 × the area of the frame.

Question 22. The correct answer is K. As the following diagrams show, there are many different arrangements of points that satisfy the conditions. But, in all of these, the order of points starting from point *A* is *A*, *B*, *D*, *C*.



Because *D* is between *C* and *B* distance *CD* is always shorter than distance *BC*. So, answer choice **K** is always true.

The other answer choices can each be true for particular arrangements of the points, but they can also each be false for particular arrangements. Line 1 shows that **F** is sometimes false, Line 2 that **G** is sometimes false, Line 3 that **H** is sometimes false, and Line 4 that **J** is sometimes false. If you got an incorrect answer, you probably did not consider enough cases.

Question 23. The correct answer is E. If you find which of the following integers could be y when the greatest common factor of x^2y^2 and xy^3 is 45, then you first recognize that the greatest common factor of x^2y^2 and xy^3 is xy^2, which equals 45. Because x and y are both positive integers and $45 = 3^25$, there are two possibilities for x, 5 or 45. If x is 45, then y is 1. If x is 5, then $y = 3$. Thus, 3 is the answer choice.

If you chose **B**, you probably found x rather than y. If you chose **C**, you probably found y^2 rather than y. If you chose **D**, you probably found xy rather than y.

MATHEMATICS • PRACTICE TEST 2 • EXPLANATORY ANSWERS

Question 24. The correct answer is G. The company's testing was randomized because the 150 volunteers were chosen without bias. The testing was also an experiment because the results from a control group (the group that received the placebo) and a treatment group (the group that received the new medication) were compared. If you chose **F**, you might not have realized that a census is a survey of a population and does not involve testing medicine. If you chose **H**, you might not have realized that the experiment was randomized by the simulation described, which is similar to the randomized selection of a marble from a bag of marbles. If you chose **J** or **K**, you might not have realized that, for a sample survey, all the volunteers would have received the same survey or test—there would be no control group or treatment group.

Question 25. The correct answer is D. The two signs will flash at the same time when the elapsed time is a common multiple of the signs' flash intervals, 4 seconds and 10 seconds. The least common multiple of 4 and 10 is 20, so the signs will flash at the same time after 20 seconds elapse.

If you chose **A**, you might have subtracted the 4-second flash interval from the 10-second flash interval $10 - 4 = 6$. If you chose **B**, you might have divided the sum of the 4-second flash interval and the 10-second flash interval by 2, $\frac{4+10}{2} = \frac{14}{2} = 7$. If you chose **C**, you might have added the 4-second flash interval to the 10-second flash interval. If you chose **E**, you might have multiplied the 4-second flash interval and the 10-second flash, $4(10) = 40$.

Question 26. The correct answer is K. Please recall that $|x|$ means the *absolute value of* x and, by definition, $|x| = x$ if $x \geq 0$ and $|x| = -x$ if $x < 0$. In any case, $|x|$ is positive for all nonzero values of x. Because the sum of 2 positive numbers is positive, $|a| + |b| > 0$. Multiplying both sides of this inequality by -1 and remembering to switch the direction of the inequality sign, we get $-(|a| + |b|) = -|a| - |b| < -0 = 0$. Therefore, the expression in **K** is *always* negative. We now show by example that the expressions in **F**, **G**, and **J** *can be* positive and that the expression in **H** is *always* positive. If you let $a = b = -1$, then $-a - b = -(-1) - (-1) = 1 + 1 = 2$. Hence, the expression in **G** can be positive. If you let $a = 2$ and $b = 1$, then $a - b = 2 - 1 = 1$, so the expression in **F** can be positive. Using those same two values for a and b, $|a| - |b| = |2| - |1| = 2 - 1 = 1$, so the expression in **J** can be positive. Finally, because the sum of two positive numbers is positive, $|a| + |b|$ is positive, so the expression in **H** is *always* positive.

Question 27. The correct answer is C. You can sketch the graphs of the two conics to determine the number of points of intersection. The circle is centered at the origin and has a radius of 3. The vertex of the parabola is $(-3,-2)$, and passes through $(-2,-1)$ so that the parabola turns upward. The vertex of the parabola is at a distance of $\sqrt{(-3)^2 + 2^2} = \sqrt{13}$ coordinate units from the origin and all points on the circle are at a distance of 3 coordinate units from the origin, so the vertex lies outside the circle. Thus, for $x < -3$, the parabola will not intersect the circle in any points. For $x > 3$, the parabola intersects the circle at two distinct points, one on the lower semicircle and one on the upper

semicircle. If you chose **A**, you may have considered the portion of the parabola determined only by $x < -3$. If you chose **B**, you may have considered only where the parabola intersects the upper semicircle. If you chose **D**, you may have thought the parabola intersects the circle at the y-intercept of the lower semicircle and passed through the upper semicircle at two distinct points. If you chose **E**, you may have thought the vertex of the parabola lies below the y-intercept of the lower semicircle circle and thus intersects both the lower semicircle and the upper semicircle in two distinct points.

Question 28. The correct answer is H. Using *of* as times and *is* as equals, you can write $0.4(250) = 0.6x$ so that $x = \frac{0.4(250)}{0.6} = 166\frac{2}{3}$. If you chose **F**, you may have incorrectly found 60% of 250. If you chose **G**, you may have incorrectly found 160% of 40% of 250. If you chose **J**, you may have incorrectly subtracted 40 from 250 and then added 60. If you chose **K**, you may have solved the incorrect equation, $0.6(250) = 0.4x$.

Question 29. The correct answer is A. We will write this inequality equivalently, with x on the left, in two steps. First, we add $6y$ to both sides and obtain $-2x > 8y - 4$. Then we divide both sides by -2, remembering to reverse the direction of the inequality sign when we do so. Doing this, we get the inequality $x < -4y + 2$, which is **A**. If you chose **B**, you probably forgot to reverse the direction of the inequality sign when you divided both sides by a negative number. If you chose **C**, you may have subtracted $6y$ from both sides instead of adding it in the first step. If you chose **D**, you may have obtained $-2x > 8y - 4$ but then missed the sign on y when you divided $8y$ by -2. If you chose **E**, you may have missed the sign on y when you divided $8y$ by -2 and also not reversed the direction of the inequality sign when you divided both sides by a negative number.

Question 30. The correct answer is F. You can use the identity $\tan\alpha = \frac{\sin\alpha}{\cos\alpha}$ or $\cos\alpha = \frac{\sin\alpha}{\tan\alpha}$ to find that $\cos\alpha = \frac{\left(\frac{40}{41}\right)}{\left(\frac{40}{9}\right)} = \frac{9}{41}$. Alternatively, you can use the ratios $\sin\alpha = \frac{\text{opposite}}{\text{hypotenuse}} = \frac{40}{41}$, $\tan\alpha = \frac{\text{opposite}}{\text{adjacent}} = \frac{40}{9}$, and $\cos\alpha = \frac{\text{adjacent}}{\text{hypotenuse}} = \frac{9}{41}$. If you chose **G**, you may have incorrectly used $\frac{\tan\alpha}{\sin\alpha}$. If you chose **H**, you may have incorrectly used $\frac{1}{\tan\alpha}$. If you chose **J** or **K**, you may have attempted to calculate the length of the hypotenuse of the triangle using the given values.

Question 31. The correct answer is B. You can use the formula for the perimeter of a rectangle, $P = 2(w + l)$ with $P = 96$, $AB = w$, and $BC = l$, to find that $w + l = \frac{96}{2} = 48$. Using this equation and the given ratio with k as the constant of proportionality, we have $w = 3k$ and $l = 5k$, so that the equation becomes $3k + 5k = 48$ or $8k = 48$. Solving for k gives $k = 6$ so that $w = AB = 3(6) = 18$ cm. If you chose **A**, you may have incorrectly thought that the length is given by the constant of proportionality. If you chose **C**, you may have incorrectly found BC rather than AB. If you chose **D**, you may have incorrectly thought the perimeter is given by $P = (w + l)$ and used this result to find AB. If you chose **E**, you may have incorrectly thought the perimeter is given by $P = (w + l)$ and used this result to find BC.

MATHEMATICS • PRACTICE TEST 2 • EXPLANATORY ANSWERS

Question 32. The correct answer is G. The area of a triangle is found by multiplying $\frac{1}{2}$ by the length of its altitude, h, and by the length of its base, b, $\frac{1}{2}hb$. For ΔABC, the length of the altitude is 8 inches and the length of the base is 16 inches. Therefore, the area of ΔABC is 64 square inches, $\frac{1}{2}(8)(16) = \frac{1}{2}(128) = 64$. The area, in square inches, of a square with side length x inches is x^2, $x^2 = 64 \Leftrightarrow x = \sqrt{64} \Leftrightarrow x = 8$. Thus the length of a side of the square is 8 inches.

If you chose **F**, **H**, **J**, or **K**, you might be misremembering the formulas for calculating the areas of a triangle and a square. If you chose **F**, you might have divided the sum of the lengths of the base and the altitude by 4, (8 + 16) ÷ 4. If you chose **H**, you might have divided the product of the length of the base and 3 by 4, (16)(3) ÷ 4. If you chose **J**, you might have correctly calculated the area of the triangle but then divided the area by 4 rather than calculating the square root, $\frac{1}{2}(8)(16) \div 4$. If you chose **K**, you might have divided the product of the lengths of the base and altitude by 4, (8)(16) ÷ 4.

Question 33. The correct answer is B. You can use the fact that $EFGH$ is a square so $HG = EF = 3.6$ meters, and $ABCD$ is a rectangle with $\overline{AD}$ and $\overline{BC}$ as opposing sides so $AD = BC = 12$ meters. The ratio of the length of $\overline{EH}$ to the length of $\overline{AD}$ is $\frac{3.6}{12} = 0.3$, so the length of $\overline{EH}$ is $0.3(100)\% = 30\%$ percent of the length of $\overline{AD}$. If you chose **A**, you may have incorrectly added the two lengths and written the result as a percent. If you chose **C**, you may have incorrectly used the ratio of the length of $\overline{EH}$ to the length of $\overline{AB}$. If you chose **D**, you may have incorrectly multiplied the two lengths and written the result as a percent.

Question 34. The correct answer is F. You can form a right triangle using $\overline{AD}$ and $\overline{AJ}$ as the legs and $\overline{JD}$ as the hypotenuse. The length of $\overline{AD}$ is 12 meters, and the length of $\overline{AJ}$ is $\frac{10}{2} = 5$ meters. By the Pythagorean theorem, $AD = \sqrt{12^2 + 5^2} = 13$ meters. If you chose **G**, you may have incorrectly added the two lengths 12 meters and 3.6 meters. If you chose **H**, you may have incorrectly added the two lengths $\frac{10}{2} = 5$ meters and 12 meters. If you chose **J**, you may have incorrectly used 12 and 10 for the lengths of the two legs and then subtracted their squares instead of adding them when using the Pythagorean theorem. If you chose **K**, you may have incorrectly used 12 and 10 for the lengths of the two legs.

Question 35. The correct answer is B. Using the fact that the circumference of a circle is π times the diameter of the circle, you can compute the length of arc $\overset{\frown}{CD}$ by finding $\frac{1}{2}$ of the circumference of the circle centered at K with radius $CK = \frac{10}{2} = 5$ meters: $\frac{1}{2}(10\pi) = 5\pi$ meters. If you chose **A**, you may have incorrectly used the radius of the circle instead of its diameter in the formula for the circumference. If you chose **C**, you may have incorrectly attempted to use the formula for the area of the circle. If you chose **D**, you may have forgotten to divide the circumference by 2. If you chose **E**, you may have incorrectly used the area of the circle.

Question 36. The correct answer is H. You can note that the y-coordinate of E is 12 because E lies on $\overline{AB}$, and $\overline{AB}$ lies on the line $y = 12$. Segment $\overline{EH}$ is perpendicular to $\overline{AB}$ and has length 3.6 coordinate units. Therefore, H has y-coordinate $12 - 3.6 = 8.4$. If you chose **F**, you may have incorrectly used $\frac{1}{2}$ of the length of $\overline{EH}$. If you chose **G**, you may have incorrectly used the length of $\overline{EH}$. If you chose **J**, you may have incorrectly used the length of $\overline{AB}$. If you chose **K**, you may have incorrectly thought that H is on the line $y = 12$.

Question 37. The correct answer is A. The length can be found by finding the positive difference in the y-coordinate at C, 4, and the y-coordinate of the point where the altitude intersects $\overline{AB}$. All ordered pairs on $\overline{AB}$ have y-coordinate 1, so the length of the altitude is $4 - 1 = 3$ coordinate units. If you chose **B**, you may have incorrectly subtracted the x-coordinate at C from the x-coordinate at B. If you chose **C**, you may have incorrectly used the length of $\overline{AB}$. If you chose **D**, you may have incorrectly used the length of $\overline{AC}$. If you chose **E**, you may have incorrectly used the midpoint of $\overline{AB}$ and found the distance from this point to C.

Question 38. The correct answer is J. The correct approach for this item is to substitute 2 for n in the formula $P = \dfrac{3^n e^{-3}}{n!}$ and then evaluate. Substituting, we get $P = \dfrac{3^2 e^{-3}}{2!}$. Because $3^2 = 9$, $2! = 2$, and $e^{-3} \approx 0.05$, we get $P \approx \frac{9(0.05)}{2} = 0.225$. The *closest* choice is **J**, 0.23. If you chose **F**, you may have forgotten to square the 3 when you evaluated. If you chose **G**, you may have thought $2! = 2^2 = 4$. Remember, by definition, $k! = 1 \times 2 \times 3 \times \cdots \times (k-1) \times k$. If you chose **H**, you may have substituted 1 for n in the formula and forgotten to square 3. Please note that the question asks for the probability that exactly 2 customers are in line, so n is 2. If you chose **K**, you may have thought $2! = 1$. Please see the previously given definition of $k!$.

Question 39. The correct answer is B. You can use the fact that the amplitude of a function of the form $g(x) = A\cos(Bx + C)$ is $|A|$. Alternatively, you can use the fact that $-1 \le \cos(3x + \pi) \le 1$ so that $-\frac{1}{2} \le \frac{1}{2}\cos(3x + \pi) \le \frac{1}{2}$. The amplitude is then given by $\frac{\text{maximum} - \text{minimum}}{2} = \frac{\frac{1}{2} - (-\frac{1}{2})}{2} = \frac{1}{2}$. If you chose **A**, perhaps you incorrectly thought the amplitude of $g(x) = A\cos(Bx + C)$ is given by $|\frac{1}{B}|$. If you chose **C**, you may have incorrectly thought the amplitude of $g(x) = A\cos(Bx + C)$ is given by $|AB|$. If you chose **D**, perhaps you incorrectly thought the amplitude of $g(x) = A\cos(Bx + C)$ is given by $|\frac{1}{A}|$. If you chose **E**, you may have incorrectly thought the amplitude of $g(x) = A\cos(Bx + C)$ is given by $|B|$.

Question 40. The correct answer is F. The Fundamental Counting Principle states that if event A can occur in m ways, and for each of these m ways event B can occur in n ways, there are exactly $m \times n$ ways both events can occur together. In making a license plate for this state, we have to make six decisions. We have to choose a letter, another letter, another letter, a digit, another digit, and one more digit. There are 26 ways to choose a letter. Because

duplications are allowed, there are 26 ways to choose the second letter, and 26 ways to choose the third letter. Similarly, there are 10 ways to choose the first digit, 10 ways to choose the second digit, and, finally, 10 ways to choose the third digit. Using the Fundamental Counting Principle, there will then be $(26)(26)(26)(10)(10)(10) = 10^3 \cdot 26^3$ distinct license plates in this state. That is **F**. **G** would be the number of ways to choose a letter *or* a digit 3 times. If you chose **H**, you might not have remembered that $k! = 1 \cdot 2 \cdot 3 \cdot 4 \cdot \ldots \cdot k$. **J** would be the number of ways of choosing 1 out of 6 items 36 times. Many permutation problems have factorials in their solutions (**K**), but 26! would mean we are arranging all 26 letters of the alphabet without allowing duplications. Similarly, 10! would mean we are arranging all 10 digits without allowing duplications.

Question 41. The correct answer is D. The median of a data set is the middle term when the set is arranged in numerical order. The 20 quiz scores are placed in score intervals that are in numerical order. Because there are 20 quiz scores, the median of the scores will be between the 10th and 11th quiz scores. There are 9 quiz scores in the 76–80 score interval, which means the 10th and 11th quiz scores occur in the 81–85 score interval. Therefore, the median of the scores is contained in the 81–85 score interval.

If you chose **A**, you might have thought the median of the scores was contained in the highest score interval. If you chose **B**, you might have thought the median of the scores was contained in the interval with the lowest frequency of scores. If you chose **C**, you might have thought the median of the scores was the middle score interval. If you chose **E**, you might have thought the median of the scores was contained in the score interval with the highest frequency of scores.

Question 42. The correct answer is J. To find an equivalent expression for $\frac{1}{1+i} \cdot \frac{1-i}{1-i}$, you multiply and get $\frac{1(1-i)}{(1+i)(1-i)} = \frac{1-i}{1-i^2} = \frac{1-i}{2}$.

If you chose **G**, you probably thought $\frac{1}{1+i}$ was equivalent to $1 + i$ and canceled $(-i)$ in both places. If you chose **H**, you probably simplified $1 - i^2$ as 1 and got $\frac{1-i}{1}$ or $1 - i$.

Question 43. The correct answer is B. For the temperature we are looking for, $F = C = x$. Substituting x for both C and F into the given formula, we get $x = \frac{9}{5}x + 32$. Multiplying both sides of this equation by 5, combining like terms, and solving for x, we get $5x = 9x + 160 \Leftrightarrow -4x = 160 \Leftrightarrow x = -40$, **B**. If you chose **A**, you may have incorrectly thought that $\frac{9}{5}$ must be distributed over $x + 32$. If you chose **C**, you may have incorrectly thought the respective freezing temperatures of water must give the desired value of x. Because water freezes at 32°F, or 0°C, you may have concluded that x must be 32°F lower than the freezing temperature in degrees Celsius. If you chose **D**, you may have tried to solve the equation $x = \frac{9}{5}x + 32$ by inspection and incorrectly thought that because there is an x-term on both sides, $x = 0$ must be the solution. If you chose **E**, you may have incorrectly thought the desired value for x is the value of F when $C = 0$.

Question 44. The correct answer is G. You can find the value closest to k by considering that the ratio of y to x is approximately k, or $\frac{y}{x} \approx k$. As can be verified, you can choose any two of the xy-pairs to find the value among the choices closest to k. For example, $\frac{0.425}{8.50} = 0.05$. If you chose **F** or **J**, you may have incorrectly found the difference in an xy-pair. If you chose **H**, you may have incorrectly multiplied an xy-pair. If you chose **K**, you may have incorrectly divided x by y.

Question 45. The correct answer is A. The correct graphical model can be identified by considering the slope and the y-intercept of the graph of the equation $2x - 5y = -5$. Writing this equation in slope-intercept form, we get $y = \frac{2}{5}x + 1$. From this, we see the slope is $\frac{2}{5}$ and the y-intercept is 1. From the y-intercept alone, we can rule out **C**, **D**, and **E** because the y-intercepts of those graphs appear to be 0, 5, and 5, respectively. The graph in **B** appears to pass close to the point (2,6). Using that point and (0,1), along with slope in terms of the change in y over the change in x, the slope of the graph in **B** is approximately $\frac{6-1}{2-0} = \frac{5}{2}$. However, the graph in **A** appears to pass close to the point (5,3). Again, using that point and (0,1), along with slope in terms of the change in y over the change in x, the slope of the graph in **A** is approximately $\frac{3-1}{5-0} = \frac{2}{5}$. Because one of the graphs must be the correct model, it must be the graph in **A**.

Question 46. The correct answer is J. To figure out how many cups of flour Diana will use, you can set up a proportion (note: $1\frac{1}{2} = 1.5$, $2\frac{1}{2} = 2.5$, and $2\frac{1}{4} = 2.25$):
$\frac{\text{teaspoons of yeast in the recipe}}{\text{cups of flour in the recipe}} = \frac{\text{teaspoons of yeast to be used}}{\text{cups of flour to be used } (x)} \Leftrightarrow$
$\frac{1.5}{2.5} = \frac{2.25}{x} \Leftrightarrow 1.5x = 2.25(2.5) \Leftrightarrow 1.5x = 5.625 \Leftrightarrow x = 3.75$. Therefore, Diana will use $3\frac{3}{4}$ cups of flour.

If you chose **F**, you might have thought the difference between the amount of yeast the recipe calls for and the amount of yeast Diana will use should be multiplied by the amount of flour the recipe calls for, $(2\frac{1}{4} - 1\frac{1}{2})(2\frac{1}{2}) = (\frac{3}{4})(\frac{5}{2}) = \frac{15}{8}$. If you chose **G**, you might have thought the difference between the amount of yeast the recipe calls for and the amount of yeast Diana will use should be added to amount of flour the recipe calls for, $(2\frac{1}{4} - 1\frac{1}{2}) + 2\frac{1}{2} = \frac{3}{4} + 2\frac{1}{2} = 3\frac{1}{4}$. If you chose **H**, you might have thought the quotient of dividing the amount of yeast Diana will use by the amount of yeast the recipe calls for should be multiplied by 2 and added to $\frac{1}{2}$, $(2\frac{1}{4} \div 1\frac{1}{2})(2) + \frac{1}{2} = (1\frac{1}{2})(2) + \frac{1}{2} = 3 + \frac{1}{2} = 3\frac{1}{2}$. If you chose **K**, you might have thought the quotient of dividing the amount of yeast Diana will use by the amount of yeast the recipe calls for should be added to the amount of flour the recipe calls for, $(2\frac{1}{4} \div 1\frac{1}{2}) + 2\frac{1}{2} = 1\frac{1}{2} + 2\frac{1}{2} = 4$.

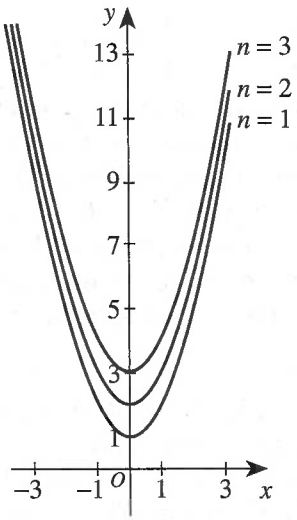
MATHEMATICS • PRACTICE TEST 2 • EXPLANATORY ANSWERS

Question 47. The correct answer is E. There are two good approaches to this problem. We will show both. First, we factor the greatest common factor out of the numerator and then cancel factors of the numerator with identical factors of the denominator. Doing so, we get $\frac{12x^6-9x^2}{3x^2}=\frac{3x^2(4x^4-3)}{3x^2}=\frac{\cancel{3x^2}(4x^4-3)}{\cancel{3x^2}}=4x^4-3$. A second approach is to use the property that states $\frac{a+b}{c}=\frac{a}{c}+\frac{b}{c}$ and then reduce both rational expressions individually. Using this approach and remembering to subtract exponents when reducing a quotient of powers, we get $\frac{12x^6-9x^2}{3x^2}=\frac{12x^6}{3x^2}-\frac{9x^2}{3x^2}=\frac{\cancel{12}}{\cancel{3}}x^{6-2}-\frac{\cancel{9}}{\cancel{3}}x^{2-2}=4x^4-3$. In either case, the correct answer is **E.** If you chose **A**, you may have used the second approach shown previously but incorrectly divided the exponents rather than subtracting them. If you chose **B**, you may have used the second approach shown previously but incorrectly divided the exponents rather than subtracting them in the first term. If you chose **C**, you may have tried the first approach but incorrectly canceled the $3x^2$ in the denominator with the identical factors of the *first term* in the numerator. Remember, you must factor the numerator first and then cancel identical factors. If you chose **D**, you may have used the second approach shown but incorrectly thought $x^{2-2}=x^0=x$. Remember, any nonzero value raised to the zero power is equal to 1.

Question 48. The correct answer is K. The dimensions of a matrix can be written as $(r \times c)$, where r is the number of rows of the matrix and c is the number of columns of the matrix. To find the element in the i^{th} row and the j^{th} column of the product of two matrices, one must multiply the elements of the i^{th} row of the matrix on the left with the corresponding elements in the j^{th} column of the matrix on the right and then add those products together. For this reason, in order for the matrix product AB to be defined, the number of columns of matrix A must be equal to the number of rows of matrix B. Another way of saying that is the product of an $(r \times c)$-matrix (on the left) and an $(m \times n)$-matrix (on the right) is defined if and only if $c = m$. (Please remember that *left* and *right* are important here because matrix multiplication is not commutative.) The dimensions of W and X are (2×2), the dimensions of Y are (2×3), and the dimensions of Z are (3×2). Because X is a (2×2)-matrix and W is a (2×2)-matrix, the matrix product XW is defined (**J**). Because X is a (2×2)-matrix and Z is a (3×2)-matrix, the matrix product XZ is undefined (**K**). Because W is a (2×2)-matrix and X is a (2×2)-matrix, the matrix product WX is defined (**F**). Because W is a (2×2)-matrix and Y is a (2×3)-matrix, the matrix product WY is defined (**G**). Because Y is a (2×3)-matrix and Z is a (3×2)-matrix, the matrix product YZ is defined (**H**). Therefore, we see that XZ, **K**, is the only indicated matrix product that is undefined.

Question 49. The correct answer is A. All the parabolas open upward. This rules out answer choices **D** and **E**. All the parabolas have the same y-intercept, (0,1). This rules out answer choice **C**, which has y-intercept equal to n, which varies. The parabolas in the family go up more quickly as the value of n increases. This means the coefficient of x must get larger as n gets larger. That happens in **A** but not in **B**.

The most common incorrect answer is **C**. A graph of that family is shown in the following figure:



Question 50. The correct answer is F. To find the minimum number of students in a class of 20 who play both guitar and piano when 8 play guitar and 9 play piano, you must look at the range for which the guitar and piano players may overlap. For instance, all 8 who play guitar could also play piano; that's the maximum. The minimum would be the smallest overlap they can have. In this case, there are 20 total; the two groups could be disjoint because 9 + 8 or 17 is less than 20. Thus, the minimum is 0.

If you chose **G**, you probably found the difference between 9 and 8. If you chose **H**, you probably found the maximum rather than the minimum number. If you chose **K**, you probably added 9 and 8.

Question 51. The correct answer is B. The sum of any two different numbers between 1 and 18 must be between 3 and 35. The only perfect squares between 3 and 35 are 4, 9, 16, and 25. Therefore the sum of each of the nine pairs must be 4, 9, 16, or 25. So far, we know the possible pairs for 1 are 3, 8, or 15 and must consider each case.

Because 16, 17, and 18 are each greater than or equal to 16, we must pair them with a number so that the sum is 25. Therefore, 16 must be paired with 9, 17 with 8, and 18 with 7. After pairing these, we are left with 1 – 6 and 10 – 15.

We now must consider what happens if 1 is paired with 3 or with 15.

If 1 is paired with 3, then 15 would have to pair with 10, 2 would have to pair with 14, 11 with 5, and 12 with 4. We'd then be left with 13 and 6 which CANNOT be paired. For that reason 1 CANNOT be paired with 3. Thus 1 *must* be paired with 15.

When 1 is paired with 15, the pairs are as follows:

$18 + 7 = 25$
$17 + 6 = 25$
$16 + 5 = 25$
$15 + 1 = 16$
$14 + 2 = 16$
$13 + 3 = 16$
$12 + 4 = 16$
$11 + 5 = 16$
$10 + 6 = 16$

If you chose **A**, you might have multiplied 1 and 16 rather than adding them together. If you chose **C**, you might have multiplied 1 and 9 and not considered that you need 9 pairs of numbers. If you chose **D** or **E**, you might not have considered that you need 9 pairs of numbers.

Question 52. The correct answer is J. Let p = the number of pennies, n = the number of nickels, d = the number of dimes, and q = the number of quarters. Each of these variables can be expressed in terms of a common variable—in this case, p.

Lucky had p pennies, so the value of the pennies was p cents. Because she had twice as many nickels as pennies, $n = 2p$, and the value of the nickels was $5(2p)$ cents. She had 1 fewer dime than nickels, so $d = n - 1 = 2p - 1$, and the value of the dimes was $10(2p - 1)$ cents. Finally, she had 1 more quarter than nickels, so $q = n + 1 = 2p + 1$, and the value of the quarters was $25(2p + 1)$ cents.

Because the total value of the coins is \$8.25, or 825 cents, p satisfies the following equation:

$$p + 5(2p) + 10(2p - 1) + 25(2p + 1) = 825$$

Multiplying out each factor on the left-hand side and collecting like terms, you can see that $81p + 15 = 825$, so $81p = 810$. Therefore, $p = 10$ and $q = 2p + 1 = 2(10) + 1 = 21$ quarters.

Question 53. The correct answer is C. Solve for x:

$$10^{\left(\frac{2x-1}{x}\right)}=1$$

$$\Leftrightarrow \log\left(10^{\left(\frac{2x-1}{x}\right)}\right)=\log 1$$

$$\Leftrightarrow \frac{(2x-1)}{x}=0$$

$$\Leftrightarrow x\left(\frac{(2x-1)}{x}\right)=0\cdot x$$

$$\Leftrightarrow 2x-1=0$$

$$\Leftrightarrow 2x=1$$

$$\Leftrightarrow x=\frac{1}{2}$$

If you chose **A**, **B**, **D**, or **E**, you might have performed an incorrect algebraic manipulation and neglected to test your calculated value of x in the equation to see if it held.

For **A**, you might have correctly manipulated the expression except for incorrectly keeping the negative on the 1:

$$10^{\left(\frac{2x-1}{x}\right)}=1$$

$$\Leftrightarrow \log\left(10^{\left(\frac{2x-1}{x}\right)}\right)=\log 1$$

$$\Leftrightarrow \frac{(2x-1)}{x}=0$$

$$\Leftrightarrow x\left(\frac{(2x-1)}{x}\right)=0\cdot x$$

$$\Leftrightarrow 2x-1=0$$

$$\Leftrightarrow 2x=-1$$

For **B**, you might have incorrectly tried to solve the exponential equation by multiplying both sides by 10 rather than taking the log of both sides:

$$10^{\left(\frac{2x-1}{x}\right)}=1$$

$$\Leftrightarrow 10\left(10^{\left(\frac{2x-1}{x}\right)}\right)=1\cdot 10$$

$$\Leftrightarrow \frac{(2x-1)}{x}=10$$

For **D**, you might have incorrectly tried to solve the exponential equation by dividing both sides by 10 rather than taking the log of both sides:

$$10^{\left(\frac{2x-1}{x}\right)}=1$$

$$\Leftrightarrow \left(10^{\left(\frac{2x-1}{x}\right)}\right)\div 10=1\div 10$$

$$\Leftrightarrow \frac{(2x-1)}{x}=\frac{1}{10}$$

For **E**, you might have incorrectly thought log1 = 1 rather than 0:

$$10^{\left(\frac{2x-1}{x}\right)}=1$$

$$\Leftrightarrow \log\left(10^{\left(\frac{2x-1}{x}\right)}\right)=\log 1$$

$$\Leftrightarrow \frac{(2x-1)}{x}=1$$

MATHEMATICS • PRACTICE TEST 2 • EXPLANATORY ANSWERS

Question 54. The correct answer is G. According to the table, 90 people play an instrument but only 50 of those 90 people like to read. Therefore, the desired probability is $\frac{50}{90} = \frac{5}{9}$.

If you chose **F**, you calculated the probability that a person randomly selected out of *the total people surveyed* both likes to read and plays a musical instrument, $\frac{50}{250}$, rather than the probability that a person randomly selected out of *the total people that play an instrument* likes to read. If you chose **H**, you calculated the probability that a person randomly selected out of *the total people that like to read* plays a musical instrument, $\frac{50}{110}$, rather than the probability that a person randomly selected out of *the total people that play an instrument* likes to read. If you chose **J**, you calculated the probability that a person randomly selected out of *the total people surveyed* plays a musical instrument, $\frac{90}{250}$, rather than the probability that a person randomly selected out of *the total people that play an instrument* likes to read. If you chose **K**, you calculated the probability that a person randomly selected out of *the total people surveyed* likes to read, $\frac{110}{250}$, rather than the probability that a person randomly selected out of *the total people that play an instrument* likes to read.

Question 55. The correct answer is A. Mario will travel a distance equal to 1 circumference of the wheel for each complete revolution of the wheel. So first, we must find the circumference of Mario's wheel. The formula for the circumference of a circle is $C = \pi D$, where D is the diameter of the circle. Hence, $C = 26\pi$ inches. Using unit multipliers to find Mario's speed in *feet per second*, we get $\left(\frac{200 \cancel{\text{rev}}}{1 \cancel{\text{min}}}\right)\left(\frac{1 \cancel{\text{min}}}{60 \text{ sec}}\right)\left(\frac{26\pi \cancel{\text{in}}}{1 \cancel{\text{rev}}}\right)\left(\frac{1 \text{ ft}}{12 \cancel{\text{in}}}\right) = \frac{65\pi \text{ ft}}{9 \text{ sec}}$. Therefore, the correct answer is **A**. If you chose **B**, you may have used the radius rather than the diameter when you found the circumference. If you chose **C**, you may have had an extra factor of 2 in the formula for circumference. If you chose **D**, you may have used the formula for area of a circle rather than the formula for circumference. If you chose **E**, you may have used the formula for area of a circle with an incorrect value for the radius rather than the formula for circumference.

Question 56. The correct answer is K. You can use properties of exponents and the fact that for $b > 0$, $b \neq 1$, $b^x = b^y \rightarrow x = y$ to solve for the value of $\frac{j}{k}$. Because $(\sqrt{3})^j = (3^{\frac{1}{2}})^j = 3^{\frac{j}{2}}$ and $27^k = (3^3)^k = 3^{3k}$, you can rewrite the equation as $3^{\frac{j}{2}} = 3^{3k} \rightarrow \frac{j}{2} = 3k$ so that $\frac{j}{k} = 3(2) = 6$. If you chose **F**, you may have incorrectly thought that $\left(\sqrt{3}\right)^j = 3^{2j}$ and $27^k = 3^{\frac{k}{3}}$ and solved $2j = \frac{k}{3}$. If you chose **G**, you may have incorrectly thought that $\left(\sqrt{3}\right)^j = 3^{2j}$ and solved $2j = 3k$. If you chose **H**, you may have ignored $\sqrt{3}$ to get $3^j = 3^{3k}$ and solved $j = 3k$. If you chose **J**, you may have incorrectly written 27^k as 3^{2k} and solved $\frac{j}{2} = 2k$.

MATHEMATICS • PRACTICE TEST 2 • EXPLANATORY ANSWERS

Question 57. The correct answer is A. $\frac{3}{4}, \frac{3}{4} + d, \frac{3}{4} + 2d, \frac{3}{4} + 3d, \frac{3}{4} + 4d, \frac{3}{4} + 5d, \frac{3}{4} + 6d$. The median of these is the middle term, $\frac{3}{4} + 3d$. The mean is the sum of all the terms divided by 7. We can quickly find the sum of these terms using the formula $S_n = \frac{n(a_1 + a_n)}{2}$ where a_1 is the first term, a_n is the nth term, and n is the number of terms. The sum is thus: $S_7 = \frac{7(a_1 + a_7)}{2} = \frac{7(\frac{3}{4} + \frac{3}{4} + 6d)}{2} = \frac{7(\frac{6}{4} + 6d)}{2} = 7(\frac{3}{4} + 3d)$. Now that you have the sum, you can divide it by 7 to get the mean of the 7 terms, $\frac{7(\frac{3}{4} + 3d)}{7} = \frac{3}{4} + 3d$. Because the mean and the median are both $\frac{3}{4} + 3d$, their difference is 0.

If you chose **B**, you might have selected the first term of the sequence. If you chose **C**, you might have taken the reciprocal of the first term. If you chose **D**, you might have thought the mean of the 7 terms was 4 and the median of the 7 terms was 7, $7 - 4 = 3$. If you chose **E**, you might have thought the mean of the 7 terms was 3 and the median of the 7 terms was 7, $7 - 3 = 4$.

Question 58. The correct answer is F. Arc length can be thought of as a fractional part of the circumference. That fraction is given by $\frac{\theta}{360}$, where θ is the measure of the central angle, in degrees, intercepting the desired arc. So first we must find the measure of $\angle D$. Using the sine ratio, we see $\sin \angle D = \frac{1}{4}$, making the measure of $\angle D$ equal to $(\sin^{-1}(\frac{1}{4}))°$. The circumference of the circle is $2\pi r = 2\pi(4) = 8\pi$ cm. Therefore, the length of $\overset{\frown}{AC}$ is $\frac{\sin^{-1}(\frac{1}{4})}{360} \times (8\pi)$. Reducing $\frac{8}{360}$, we get $\frac{\pi}{45}(\sin^{-1}(\frac{1}{4}))$, which is **F**. If you chose **G**, you may have incorrectly thought that the cosine ratio, not the sine ratio, is given by the length of the opposite leg over the length of the hypotenuse. If you chose **H**, you may have incorrectly used the formula for the area of the circle rather than the formula for the circumference of the circle. If you chose **J**, you may have made *both* of the described errors. If you chose **K**, you may have incorrectly thought that the tangent ratio, not the sine ratio, is given by the length of the opposite leg over the length of the hypotenuse *and* also incorrectly used the formula for the area of the circle rather than the formula for the circumference of the circle.

Question 59. The correct answer is C. The area of the triangle can be found by using the given formula, $\frac{1}{2}ab \sin C$. Since only one angle measure is given in the triangle, use that for C in the formula: $\frac{1}{2}ab \sin 30°$. The length of the side opposite from angle C, the 30° angle, is 5.0 cm and must correspond to side c. That means that the other side lengths, 8.0 cm and 4.0 cm, correspond to sides a and b and can be plugged into the formula: $\frac{1}{2}(8.0)(4.0)(\sin 30°)$. Since the sine of 30° is $\frac{1}{2}$, $\frac{1}{2}(8.0)(4.0)(\sin 30°) = \frac{1}{2}(8.0)(4.0)(\frac{1}{2})$, which results in 8 cm^2, the area of the triangle. If you chose **A**, you may have incorrectly thought the area of the triangle was equal to $8.0(\sin 30°)$. If you chose **B**, you may have used the given formula but incorrectly used 4.0 and 5.0 for the side lengths of a and b rather than 4.0 and 8.0. If you chose **D**, you may have incorrectly thought the given triangle was a right triangle and calculated the area using $\frac{1}{2}(5.0)(4.0)$, or you may have used the given formula but incorrectly used 5.0 and 8.0 for the side lengths of a and b rather than 4.0 and 8.0. If you chose **E**, you may have incorrectly used the cosine of 30° rather than the sine of 30°.

Question 60. The correct answer is K. The expected value of a random variable is its theoretical long-run average value. We find this by

$$E(X)=\sum_{x_i} x_i P(X=x_i)=\tfrac{1}{6}(0) + \tfrac{1}{12}(1) + \tfrac{1}{4}(2) + \tfrac{1}{12}(3) + \tfrac{1}{12}(4) + 0(5) + \tfrac{1}{3}(6).$$

You may have gotten **F** if you did $(\frac{1}{6} + \frac{1}{12} + \frac{1}{4} + \frac{1}{12} + \frac{1}{12} + \frac{1}{3}) \div 6$. You may have gotten **G** if you recorded the last entry in the table. You may have gotten **H** if you did $\frac{1}{6} + \frac{1}{12} + \frac{1}{4} + \frac{1}{12} + \frac{1}{12} + 0 + \frac{1}{3}$. You may have gotten **J** if you did $\frac{1}{3}(6)$.