MATHEMATICS • PRACTICE TEST 1 • EXPLANATORY ANSWERS

**Question 1. The correct answer is D.** The average amount Xuan paid for each new book is given by $\frac{9(\$9.80)-\$37.80}{4} = \$12.60$. Multiply to find the amount from the sales, subtract to find the total amount spent on the new books, and divide to find the average spent per new book. If you answered **A**, you may have divided by 9 instead of 4: $\frac{9(\$9.80)-\$37.80}{9} = \$5.60$. If you answered **B**, you may have divided the leftover dollar amount by 4: $\frac{\$37.80}{4} = \$9.45$. If you answered **C**, you may have divided by the difference of 9 and 4: $\frac{9(\$9.80)-\$37.80}{9-4} = \$10.08$. If you answered **E**, you may have forgotten to subtract before dividing: $\frac{9(\$9.80)}{4} = \$22.05$.

**Question 2. The correct answer is H.** The coordinates of the point after the translation are given by $(-5 + 7, 7 - 5)$, which is $(2,2)$ because translating right 7 coordinate units corresponds to adding 7 to the $x$-coordinate, and translating down 5 coordinate units corresponds to subtracting 5 from the $y$-coordinate. If you answered **F**, you may have subtracted instead of adding and added instead of subtracting: $(-5 - 7, 7 + 5)$. If you answered **G**, you may have switched the 7 and 5: $(-5 + 5, 7 - 7)$. If you answered **J**, you may have added for both coordinates instead of subtracting for the downward translation: $(-5 + 7, 7 + 5)$. If you answered **K**, you may have added 5 and 7 for both coordinates: $(5 + 7, 7 + 5)$.

**Question 3. The correct answer is C.** Shantiel's average speed, in miles per hour, is given by $\frac{648}{24-(9-3)} = 36$. Divide the number of miles traveled by the elapsed time. To find the elapsed time, you could realize that 9 a.m. the next day would be 24 hours after the time she left, and 3 a.m. the next day is 6 hours before 9 a.m. the next day. If you answered **A**, you may have divided by 9 instead of 18: $\frac{648}{9} = 72$. If you answered **B**, you may have thought the elapsed time was 12 instead of 18: $\frac{648}{9+3} = 54$. If you answered **D**, you may have thought the elapsed time was 21 instead of 18: $\frac{648}{9+12} \approx 31$. If you answered **E**, you may have found the elapsed time only: $24 - (9 - 3) = 18$.

**Question 4. The correct answer is J.** The total number of text messages for which Juan was charged on this bill is given by $300 + \frac{\$16.50 - \$10.00}{\$0.10} = 365$. The bill was greater than \$10.00, so Juan sent at least 300 text messages. To find how many more than 300, subtract \$10.00 from the total charge of \$16.50 to find that he was charged \$6.50 more than the initial \$10.00, and then divide the difference by the \$0.10 charge per text message beyond the first 300. If you answered **F**, you may have subtracted from instead of adding to the first 300 messages: $300 - \frac{\$16.50 - \$10.00}{\$0.10} = 235$. If you answered **G**, you may have subtracted from instead of adding to the first 300 messages, forgotten to subtract the initial \$10.00 from the \$16.50, and divided by \$1.10 instead of \$0.10: $300 - \frac{\$16.50}{\$1.10} = 285$. If you answered **H**, you may have forgotten to subtract the initial \$10.00 from the \$16.50 and then divided by \$1.10 instead of \$0.10: $300 + \frac{\$16.50}{\$1.10} = 315$. If you answered **K**, you may have forgotten to subtract the initial \$10.00 from the \$16.50: $300 + \frac{\$16.50}{\$0.10} = 465$.

**Question 5. The correct answer is A.** The sum of the matrices is given by $\begin{bmatrix} 9+(-6) & 8+6 \\ -4+5 & 7+4 \end{bmatrix}$. Add all pairs of corresponding entries. If you answered **B**, you may have disregarded the negative in the first matrix: $\begin{bmatrix} 9+(-6) & 8+6 \\ 4+5 & 7+4 \end{bmatrix}$. If you answered **C**, you may have disregarded both negatives: $\begin{bmatrix} 9+6 & 8+6 \\ 4+5 & 7+4 \end{bmatrix}$. If you answered **D**, you may have added elements within each matrix instead of the corresponding elements: $\begin{bmatrix} 9+8 & -6+6 \\ -4+7 & 5+4 \end{bmatrix}$. If you answered **E**, you may have multiplied the matrices instead of adding: $\begin{bmatrix} 9(-6)+8(5) & 9(6)+8(4) \\ -4(-6)+7(5) & -4(6)+7(4) \end{bmatrix}$.

**Question 6. The correct answer is H.** The value of $f(4,3)$ is given by $3(4)^2 - 4(3) = 3(16) - 4(3) = 48 - 12 = 36$, which is found by substituting $x$ and $y$ with 4 and 3, respectively, and then squaring, multiplying, and subtracting. If you answered **F**, you may have mixed up the values substituted for $x$ and $y$: $3(3)^2 - 4(4) = 3(9) - 4(4) = 27 - 16 = 11$. If you answered **G**, you may have squared the 3 instead of the 4 in the first term: $3^2(4) - 4(3) = 9(4) - 4(3) = 36 - 12 = 24$. If you answered **J**, you may have mixed up the values substituted for $x$ and $y$ and multiplied before squaring: $(3(3))^2 - 4(4) = 9^2 - 16 = 81 - 16 = 65$. If you answered **K**, you may have multiplied before squaring: $(3(4))^2 - 4(3) = 12^2 - 12 = 144 - 12 = 132$.

**Question 7. The correct answer is D.** The mean age is given by $\frac{3(10) + 2(5)}{5} = 8$. If you answered **A**, you may have found the mean of the four numbers given in the first sentence after "5 children": $\frac{3 + 10 + 2 + 5}{4} = 5$. If you answered **B**, you may have mixed up the number of children at each age: $\frac{2(10) + 3(5)}{5} = 7$. If you answered **C**, you may have found the mean of the two unique ages: $\frac{10 + 5}{2} = 7.5$. If you answered **E**, you found the mode, the most frequently occurring age: 10.

**Question 8. The correct answer is H.** The value $DE$, in feet, is 4. Because $\overline{AC} \parallel \overline{DE}$, by corresponding angles, we have $\angle A \cong \angle BDE$. We know $\angle B \cong \angle B$, and so by the angle-angle postulate, we know $\Delta ABC \sim \Delta DBE$. Because $BD = AD$, we know $BD = \frac{1}{2}BA$. Therefore, $DE = \frac{1}{2}AC = \frac{1}{2}(8) = 4$. If you answered **F**, you may have subtracted 8 from 10: $10 - 8 = 2$. If you answered **G**, you may have subtracted half of 10 from 8: $8 - \frac{10}{2} = 3$. If you answered **J**, you may have taken half of 10 instead of half of 8: $\frac{10}{2} = 5$.

**Question 9. The correct answer is B.** The number expected to vote in favor of the proposal is given by the solution to the proportion $\frac{337}{500} = \frac{x}{22{,}000}$; $500x = 337(22{,}000)$; $x = \frac{337(22{,}000)}{500} = 14{,}828$. If you answered **A**, you may have rounded $\frac{337}{500}$ to $\frac{3}{5}$ and solved the proportion $\frac{3}{5} = \frac{x}{22{,}000}$. If you answered **C**, you may have subtracted the difference of 500 and 337 from 22,000: $22{,}000 - (500 - 337) = 21{,}837$. If you answered **D**, you chose the total number of registered voters. If you answered **E**, you may have solved a different proportion: $\frac{337}{500} = \frac{22{,}000}{x}$.

**Question 10. The correct answer is G.** The difference between the total of all Diego's payments and the car's purchase price is given by $\$400+48(\$300)-\$13,400=\$1,400$. To the down payment, add the product of 48 payments and the amount per payment, and then subtract the purchase price. If you answered **F**, you may have forgotten to add the down payment: $48(\$300)-\$13,400=\$1,000$. If you answered **H**, you may have subtracted the down payment from the purchase price, disregarding the monthly payments: $\$13,400-\$400=\$13,000$. If you answered **J**, you found the amount made in monthly payments: $48(\$300) = \$14,400$. If you answered **K**, you didn't subtract the purchase price $\$400 + 48(\$300) + \$14,800$.

**Question 11. The correct answer is A.** Rearrange the given equation into slope-intercept form, $y = mx + b$, where $m$ is the slope. $4x + 7y = 9 \rightarrow 7y = -4x + 9 \rightarrow y = -\frac{4}{7}x + \frac{9}{7}$. In this equation, $m = -\frac{4}{7}$. If you chose **B**, you may have divided the whole equation by 9 first and interpreted the coefficient of $x$ as the slope before rearranging the rest of the equation into slope-intercept form. If you chose **C**, you may not have divided the rearranged equation by 7 before interpreting the coefficient of $x$ as the slope. If you chose **D**, you may have interpreted the given coefficient of $x$ as the slope without rearranging the equation. If you chose **E**, you may have interpreted the right side of the equation as the slope.

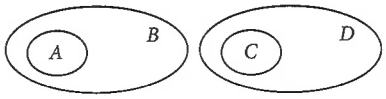
**Question 12. The correct answer is G.** Because $\angle BCD$ is supplementary to $\angle DCF$, $m\angle BCD + m\angle DCF = 180°$. To find $m\angle DCF$, use the angle sum of $\Delta DCF$, $m\angle CFD + m\angle FDC + m\angle DCF = 180°$. Notice that $m\angle CFD = m\angle GFE = 25°$ because they are vertical angles. Because $\angle FDC$ is a right angle, the angle sum is $25° + 90° + m\angle DCF = 180° \rightarrow m\angle DCF = 180° - (25° + 90°) = 65°$. Substituting $m\angle DCF = 65°$ into the first equation, $m\angle BCD + m\angle DCF = 180°$, results in $m\angle BCD + 65° = 180° \rightarrow m\angle BCD = 115°$. If you chose **F**, you may have found $m\angle DCF$ and not used its supplementary relationship with $\angle BCD$. If you chose **H** or **J**, you may have tried to estimate the measure from the figure. If you chose **K**, you may have tried to use $m\angle GFE = 25°$ as a supplementary angle to $\angle BCD$.

**Question 13. The correct answer is C.** Factoring the polynomial in the given equation results in $(x + 6)(x - 5) = 0$. The solutions of this equation are the values of $x$ that make the equation true. Because the left side of the equation is a product of two factors, it is sufficient for either factor to be zero, meaning $(x + 6) = 0$ and $(x - 5) = 0$. The solutions to these two equations are $-6$ and 5. The sum of these two solutions is $-6 + 5 = -1$. If you chose **A**, you may have thought that the coefficient $-30$ was a solution to the equation. If you chose **B** or **E**, you may have tried substituting $-6$ or 5 and found that this made the given equation true. If you chose **D**, you may have thought that the right side of the given equation was the solution.

**Question 14. The correct answer is F.** The radius of the sphere is half the diameter, so $r = 2$ yards. Substituting this into the given volume formula results in $\frac{4\pi(2)^3}{3} = \frac{4(8)\pi}{3} = \frac{32}{3}\pi$ cubic yards. If you chose **G**, you may have computed $\frac{4^3\pi}{3}$. If you chose **H**, you may have forgotten to include the denominator of the given formula in your answer. If you chose **J**, you may have multiplied the diameter by 3 and forgotten to include the denominator of the given formula. If you chose **K**, you may have substituted the diameter instead of the radius.

**Question 15. The correct answer is C.** Estimate $\sqrt{85}$ by using the known values of perfect squares $9 = \sqrt{81}$ and $10 = \sqrt{100}$. Because $9 = \sqrt{81} < \sqrt{85} < \sqrt{100} = 10$, you can conclude that $9 < \sqrt{85} < 10$, which means that 10 is the smallest integer greater than $\sqrt{85}$ because there are no integers between 9 and 10. If you chose **A**, you may have thought you were looking for the smallest digit in the number 85. If you chose **B**, you may have found that $\sqrt{85} \approx 9.2195$ and rounded down. If you chose **D**, you may have checked that 12 was larger than $\sqrt{85}$ and may not have looked for a smaller integer. If you chose **E**, you may have thought that the square root meant to divide by 2.

**Question 16. The correct answer is K.** The relationships in the three statements can be represented as shown in this figure.




Because no elements of $D$ are elements of $B$, $D \cap B = \varnothing$, it must follow that no elements of $A$ are elements of $C$, $C \cap A = \varnothing$, because $A$ and $C$ are entirely contained in $B$ and $D$, respectively, $A \subset B$ and $C \subset D$. **F** cannot be true because no elements of $A$ are elements of $C$. **G** cannot be true because no elements of $D$ are elements of $B$. **H** cannot be true because $B$ and $D$ have no common elements, and any element of $C$ is an element of $D$. **J** cannot be true because all elements of $A$ are elements of $B$.

**Question 17. The correct answer is A.** The midpoint (2,1) must also be the midpoint of the $x$- and $y$-component distances of the length of $\overline{AB}$. Because the $x$-distance from point $A$ to (2,1) is $8 - 2 = 6$, point $B$ must be 6 units from (2,1) in the $x$-direction, which is $2 - 6 = -4$. If you chose **B**, you may have found the change in the $x$-coordinate from (2,1) to point $A$. If you chose **C**, you may have found the $y$-coordinate of point $B$. If you chose **D**, you may have found half the $x$-distance from point $A$ to (2,1). If you chose **E**, you may have found the $x$-coordinate of the midpoint of (2,1) and point $A$.

**Question 18. The correct answer is K.** The total number of either pitcher or outfielder cards is $8 + 7 = 15$. The probability of finding one of these 15 cards in one pick is the fraction of desired outcomes over all possible outcomes, $\frac{15}{25} = 60\%$. If you chose **F**, you may have found the probability of each desired outcome, multiplied them, and rounded to the nearest percentage. If you chose **G**, you may have found the probability of picking an outfielder only. If you chose **H**, you may have found the probability of picking a pitcher only. If you chose **J**, you may have found the probability of picking an infielder or a pitcher.

**Question 19. The correct answer is A.** Let the change in mass of nitrogen phosphate used be $y$ and the change in the average temperature be $x$. Then $x$ and $y$ are in a proportional relationship where $y = -1.2x$. For the given change of 83.0°F − 75.0°F = 8.0°F, $y = -1.2(8.0) = -9.6$ kg. Because this is a change from a starting mass of 40.0 kg, the required application amount is 40.0 − 9.6 = 30.4 kg. If you chose **B**, you may have added 1.2 and 8.0 to find the change in mass. If you chose **C**, you may have divided 40 by 1.2. If you chose **D**, you may have subtracted the change in temperature from the mass before multiplying by 1.2. If you chose **E**, you may have found the required application amount for a change of only 1°F.

**Question 20. The correct answer is G.** The area of the shaded region is $xy$. From the figure, create the equations $6 = y + 2 + y$ and $5 = x + 2 + x$, which follow from the equivalent side lengths of the rectangle. Solve each of these equations: $6 = y + 2 + y \rightarrow 2y = 6 - 2 \rightarrow y = 2$ and $5 = x + 2 + x \rightarrow 2x = 5 - 2 \rightarrow x = \frac{3}{2}$. Then the shaded area $xy = \left(\frac{3}{2}\right)2 = 3$ square units. If you chose **F**, you may have found the value of $x$ and squared it. If you chose **H**, you may have assumed all the regions were equal in area and divided 30 by 9. If you chose **J**, you may have assumed all the regions were equal in area and that the whole figure was a square with a side length of 6 units. If you chose **K**, you may have found $2(x + y)$ instead of $xy$.

**Question 21. The correct answer is D.** The perimeter of the scalene triangle given is 100 inches. We also know that the perimeter of the triangle is the sum of the lengths of its sides. For this triangle, that means the triangle has a perimeter of $5x + (3x + 30) + (2x + 10) = 10x + 40$. Therefore, $10x + 40$ must equal 100 inches because both values are the perimeter of this triangle. Thus, $10x = 60$ and $x = 6$. Because the sides of this triangle are $5x$, $3x + 30$, and $2x + 10$, we know that the side lengths are $5(6) = 30$, $3(6) + 30 = 48$, and $2(6) + 10 = 22$. Because 48 is the largest among 30, 48, and 22, we know that the longest side of this triangle is 48 inches. If you chose **A**, you may have thought the longest side of the triangle was the value of $x$. If you chose **B**, you may have thought the longest side was the side of length $2x + 10$. If you chose **C**, you may have thought the longest side was the side of length $5x$.

**Question 22. The correct answer is J.** Because each council member gets a seat, there are 6 members that could be in Seat 1. Because someone is already in Seat 1, there are 5 options for Seat 2. Because someone is already in Seats 1 and 2, there are 4 options for Seat 3. Similarly, there are 3 options for Seat 4, 2 options for Seat 5, and 1 option for Seat 6. Thus, the number of different arrangements is $6(5)(4)(3)(2)(1) = 720$. If you chose **F**, you may have added $6 + 5 + 4 + 3 + 2 + 1 = 21$ instead of multiplying. If you chose **G**, you may have multiplied the number of council members by the number of seats available. If you chose **H**, you may have taken $\frac{6^6}{6!}$ and rounded to the nearest integer. If you chose **K**, you may have taken 6 council members and added a 0 in front of it for each seat available.

**Question 23. The correct answer is E.** 200% of 1 is 2(1) = 2. The sum of 2 and 200% of 1 is equivalent to 2 + 2 = 4. 400% of 1 is 4(1) = 4. If you chose **A**, you may have thought that 100% of 2, which equals 2, equals 4. If you chose **B**, you may have thought that 150% of 2, which equals 3, equals 4. If you chose **C**, you may have thought that 300% of 2, which equals 6, equals 4. If you chose **D**, you may have thought that 300% of 1, which equals 3, equals 4.

**Question 24. The correct answer is H.** Given two points of a line, $(x_1, y_1)$ and $(x_2, y_2)$, the slope of the line is given by $\frac{y_2 - y_1}{x_2 - x_1}$. For this line, the points given are $(0, 2)$ and $(3, 0)$. So, the slope of the line is $\frac{0-2}{3-0} = \frac{-2}{3} = -\frac{2}{3}$. The $y$-intercept of this line is $(0, 2)$. The slope-intercept form of a line is given by $y = mx + b$ where $m$ is the slope and $b$ is the $y$-value of the $y$-intercept. Therefore, the slope-intercept form of this line is $y = -\frac{2}{3}x + 2$. If you chose **F**, you may have thought that the slope of a line is given by $\frac{x_2 - x_1}{y_2 - y_1}$. If you chose **G**, you may have thought that the slope of a line is given by $\frac{x_2 - x_1}{y_2 - y_1}$ and that $b$ is the $x$-value of the $x$-intercept. If you chose **J**, you may have thought that $b$ is the $x$-value of the $x$-intercept. If you chose **K**, you may have thought that the slope of a line is given by $-\frac{y_2 - y_1}{x_2 - x_1}$.

**Question 25. The correct answer is C.** The area of a kite is given by $\frac{1}{2}$ multiplied by its length multiplied by its width. So, for this kite, the area is given by $\frac{1}{2}(40)(28) = 560$. If you chose **A**, you may have thought the area is given by 2(40)+2(28) – 5(7). If you chose **B**, you may have thought the area is given by $\frac{1}{4}$ multiplied by its length multiplied by its width. If you chose **D**, you may have thought the area is given by $\frac{3}{4}$ multiplied its length multiplied by its width. If you chose **E**, you may have thought the area was given by $\frac{7}{8}(40)(28)$.

**Question 26. The correct answer is H.** The surface area, including the top of the pool, is 60(25) = 1,500. The area of the surface of the pool is 15(40) = 600. Therefore, the area of the patio surrounding the pool is 1,500 – 600 = 900 square feet. If you chose **F**, you may have calculated the area by (60 – 40)25. If you chose **G**, you may have calculated the surface of the pool. If you chose **J**, you may have calculated the area by 25(60) – 40(5)(2). If you chose **K**, you may have calculated the area by $25(60) - 15\left(\frac{40}{4}\right)$.

**Question 27. The correct answer is D.** The cost of the materials is the cost per foot multiplied by the number of feet. Because the number of feet is the perimeter of the outer edge of the patio, the number of feet is 2(60) + 2(25) = 170. Therefore, the cost is \$12(170) = \$2,040. If you chose **A**, you may have calculated the perimeter as 60 + 25 = 85. If you chose **B**, you may have calculated the perimeter of the pool instead of the perimeter of the outside of the patio, 2(40) + 2(15) = 110. If you chose **C**, you may have calculated the perimeter as 6(25). If you chose **E**, you may have calculated the cost of a fence around the perimeter of the pool and the cost of a fence around the outside of the patio and added them together.

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**Question 28. The correct answer is J.** Johann's average speed can be calculated by dividing the total distance over the total time. The total distance is given by 5(2)(40) = 400. Therefore, the average speed is given by $\frac{400}{4.5} \approx 89$. If you chose **F**, you may have calculated the total distance by $5\left(\frac{1}{2}\right)(60)$. If you chose **G**, you may have calculated the total distance by 5(40). If you chose **H**, you may have calculated the total distance by 5(60). If you chose **K**, you may have calculated the total distance by 5(2)(60).

**Question 29. The correct answer is B.** The slope can be found as rise over run. The rise can be calculated by 4-9. The run can be calculated by 40-20-12. Therefore, the slope is $-\frac{5}{8} = -0.625$. If you chose **A**, you may have calculated $-\frac{4}{9}$. If you chose **C**, you may have calculated $-\frac{9}{12}$. If you chose **D**, you may have calculated $-\frac{12}{9}$. If you chose **E**, you may have calculated $-\frac{8}{5}$.

**Question 30. The correct answer is K.** The company will order 3(1) + 4(1) + 6(2) = 19 bathtubs, 3(0) + 4(1) + 6(1) = 10 shower stalls, and 3(1) + 4(2) + 6(4) = 35 sinks. This will cost 19($250) + 10($150) + 35($120) = $10,450. If you chose **F**, you may have calculated the number of bathtubs as 1 + 0 + 1, the number of shower stalls as 1 + 1 + 2, and the number of sinks as 2 + 1 + 4. If you chose **G**, you may have calculated the number of bathtubs as 1 + 1 + 2, the number of shower stalls as 0 + 1 + 1, and the number of sinks as 1 + 2 + 4, gotten a total of $2,140, and decided that this was close enough to $2,070. If you chose **H**, you may have calculated the number of bathtubs as 3(1 + 0 + 1), the number of shower stalls as 4(1 + 1 + 2), and the number of sinks as 6(2 + 1 + 4). If you chose **J**, you may have calculated the number of bathtubs as 3(1) + 4(0) + 6(1), the number of shower stalls as 3(1) + 4(1) + 6(2), and the number of sinks as 3(2) + 4(1) + 6(4).

**Question 31. The correct answer is C.** To calculate half the length, exchange 1 foot for 12 inches. Thus, 5 feet 6 inches is the same as 4 feet 18 inches. Then divide each number by 2, resulting in 2 feet 9 inches. If you answered **A**, you may have dropped the original 6 inches, converted 1 foot to 10 inches, and divided 4 feet 10 inches by 2. If you answered **B**, you may have converted 1 foot to 10 inches and divided 4 feet 16 inches by 2. If you answered **D**, you may have rounded up to the next foot to 6 feet and divided it by 2. If you answered **E**, you may have added 1 to the number of feet and subtracted 1 from the number of inches; then you may have divided the number of feet by 2 and left the number of inches as is, resulting in 3 feet 5 inches.

**Question 32. The correct answer is H.** The total number of feet is the sum of the first four drives: $\left(18+18\times\frac{2}{3}+12\times\frac{2}{3}+8\times\frac{2}{3}=43\frac{1}{3}\right)$. If you answered **F**, you may have added $12+8+5\frac{1}{3}+3\frac{5}{9}$. If you answered **G**, you may have added only the first two heights: 18 + 12. If you answered **J**, you may have added 18 + 12 + 18. If you answered **K**, you may have multiplied 18 × 3.

**Question 33. The correct answer is B.** To reflect a point across the $y$-axis, multiply its $x$-coordinate by $-1$ and do not change its $y$-coordinate. So $(2, -3)$becomes $(2 \times -1, 3)$, or $(-2, 3)$. If you answered **A**, you may have interchanged the coordinates. If you answered **C**, you may have multiplied both coordinates by $-1$. If you answered **D**, you may have multiplied the $y$-coordinate by $-1$. If you answered **E**, you may have interchanged the coordinates and then multiplied them both by $-1$.

**Question 34. The correct answer is K.** The mean is the sum of the four numbers divided by 4: $\frac{a+b+c+d}{4} = mean$. Adding 3 to both sides yields $\frac{a+b+c+d}{4} + 3 = mean + 3$. It follows that $\frac{a+b+c+d}{4}+3=\frac{a+b+c+d+x}{4} = mean + 3$ where $x$ is the addition to the four numbers that results in the new mean. Now $\frac{a+b+c+d}{4}+3 = \frac{a+b+c+d}{4}+\frac{12}{4}=\frac{a+b+c+d+12}{4}$, so $x = 12$. If you answered **F**, you may have divided the increase in the mean by the four numbers. If you answered **G**, you may have subtracted the increase in the mean from the four numbers. If you answered **H**, you may have divided the four numbers by the increase in the mean. If you answered **J**, you may have added the increase in the mean and the four numbers.

**Question 35. The correct answer is E.** Using laws of exponents, the expression is equivalent to $(3+x)^{100\times(-1)}=[(3+x)^{100}]^{-1}=\frac{1}{(3+x)^{100}}$. If you answered **A**, you may have distributed the $-1$ to each term and then distributed the 100 as the exponent of each term: $-1(3^{100}+x^{100})$. If you answered **B**, you may have multiplied each term by the exponent: $-100(3+x)$. If you answered **C**, you may have used the equivalent expression $\frac{1}{(3+x)^{100}}$ and then distributed the exponent to each term. If you answered **D**, you may have used the equivalent expression $\frac{1}{(3+x)^{100}}$ and then thought $3 + x = 3x$.

**Question 36. The correct answer is G.** To find a vertical asymptote, set the denominator equal to 0, $2x - 6 = 0$, and solve for $x$. One method for solving is to first divide both sides by 2, $x - 3 = 0$, so $x = 3$. If you answered **F**, you may have divided the constant term in the numerator by the constant term in the denominator. If you answered **H**, you may have set the numerator equal to 0, $3x - 12 = 0$, and divided each side by 3, $x - 4 = 0$, so $x = 4$. If you answered **J**, you may have set the denominator equal to 0, $2x - 6 = 0$, and then divided only the $x$ term by 2, $x - 6 = 0$, so $x = 6$. If you answered **K**, you may have set the numerator equal to 0, $3x - 12 = 0$, and divided only the $x$ term by 3, $x - 12 = 0$, so $x = 12$.

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**Question 37. The correct answer is C.** The equation $xy = 0$ and the zero-product rule require that either $x$ or $y$ is equal to 0 or that both $x$ and $y$ are equal to 0. The second equation $\frac{x}{y} = 0$ requires that $x$ is equal to 0 and $y$ is not equal to 0. Combining the requirements of these two equations, it follows that both $x = 0$ and $y \neq 0$ must be true. If you answered **A**, you may have considered only the first equation and misinterpreted the zero-product rule as stating that the value of both variables must be 0. If you answered **B**, you may have considered the second equation but switched the restrictions and thought $x \neq 0$ and $y = 0$. If you answered **D**, you may have considered the second equation but thought both the numerator and the denominator must not be 0. If you answered **E**, you may have tried to list all the restrictions together and obtained $y$ may be 0 or $y$ may not be 0, which is not an answer choice.

**Question 38. The correct answer is F.** To convert $\frac{3.7\ km}{15\ min}$ to $\frac{miles}{hour}$, it is necessary to multiply $\frac{3.7\ km}{15\ min} \times \frac{60\ min}{1\ hour} \times \frac{1\ mile}{1.6\ km} \approx 9.25$ miles per hour. If you answered **G**, you may have multiplied $\frac{3.7\ km}{15\ min} \times \frac{60\ min}{1\ hour}$. If you answered **H**, you may have calculated $\frac{60\ min}{15\ min}(3.7+1.6)$. If you answered **J**, you may have multiplied $\frac{3.7\ km}{15\ min} \times \frac{60\ min}{1\ hour} \times 1.6$. If you answered **K**, you may have calculated $\frac{3.7(15)}{1.6}$.

**Question 39. The correct answer is A.** One million can be written as $1{,}000{,}000 = 10^6$, so 2.5 million can be written as $2.5 \times 10^6$. If you answered **B**, you may have thought that because 1 million has 7 digits, the exponent is 7. If you answered **C**, you probably thought there are 7 digits in a million and then added 1 digit to the left in 2.5, resulting in 8 for the exponent. If you answered **D**, you may have dropped the decimal in 2.5 and used the exponent of 10 for 1 million. If you answered **E**, you may have dropped the decimal point and thought the exponent for 1 million is 7.

**Question 40. The correct answer is K.** Let $x$ be the distance along the level ground between the point adjacent to the 70° angle and the center of the base of the tower. Then, by the definition of tangent, $\tan 70° = \frac{320}{x}$ or $= \frac{320}{\tan 70°}$. If you answered **F**, you may have used the equation $\cos 70° = \frac{x}{320}$. If you answered **G**, you may have used the equation $\sin 70° = \frac{x}{320}$. If you answered **H**, you may have used the equation $\tan 70° = \frac{x}{320}$. If you answered **J**, you may have used the equation $\sin 70° = \frac{320}{x}$.

**Question 41. The correct answer is E.** Adding the vectors and setting that sum equal to $6\mathbf{i} - 6\mathbf{j}$ results in $(a\mathbf{i}+3\mathbf{j})+(-2\mathbf{i}+b\mathbf{j})=(6\mathbf{i}-6\mathbf{j})$. Equating the coefficients on each side for **i** and **j** separately results in $a - 2 = 6$ or $a = 8$ and $3 + b = -6$ or $b = -9$. If you answered **A**, you may have solved the equations correctly but set the variables equal to the other variable's value. If you answered **B**, you may have solved the correct equations but used the wrong sign in the resulting values. If you answered **C**, you may have solved $a - 2 = -6$ and $3 + b = 6$. If you answered **D**, you may have solved $a + 2 = 6$ and $3 - b = 6$.

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**Question 42. The correct answer is F.** Subtracting 50 from both sides results in $c-50=10b^3$. Dividing by 10 on both sides results in $\frac{c}{10} - 5 = b^3$. Taking the third root of both sides and swapping the two expressions results in $b=\left(\frac{c}{10}-5\right)^{\frac{1}{3}}$. If you answered **G**, you may have put the 50 on the left side without changing the sign and then divided and took the third root as shown. If you answered **H**, you may have subtracted the 50 and divided by 10, but when taking the third root was done, the third root of the $\frac{1}{10}$ was not done. If you answered **J**, you may have divided only the 50 and the 10 by 10, resulting in $c = b^3 + 5$, then added the 5 on the left side of the equation and subtracted it from the left, and then switched the exponent from $b$ to $c$, resulting in $c^3 + 5 = b$. If you answered **K**, you may have switched the $b$ and $c$.

**Question 43. The correct answer is A.** Substituting $x + 1$ for x in $f(x)$ results in $(x+1)^2+3(x+1)=x^2+2x+1+3x+3$ or $x^2 + 5x + 4$. If you answered **B**, you may have substituted $x^2 + 3x$ in $g(x)$, resulting in $(x^2 + 3x) + 1$ or $x^2 + 3x + 1$. If you answered **C**, you may have substituted $x(x+1)^2+3x(x+1)=x^3+2x^2+x+3x^2+3x=x^3+25+4x$. If you answered **D**, you may have substituted $x^2(x+1)+3x(x+1)=x^3+x^2+3x^2+3x=x^3+4x^2+3x$. If you answered **E**, you may have substituted $x^3(x+1)+3x^2(x+1)=x^4+x^3+3x^3+3x^2=x^4+4x^3+3x^2$.

**Question 44. The correct answer is J.** The diameter of the second circle is found by multiplying 12 by 1.25, resulting in 15. Dividing the diameters by 2 to get the radius of each results in 6 and 7.5. Using the area of a circle with radius $r$ formula $\pi r^2$ results in the areas of the two circles being $36\pi$ and $56.25\pi$. The difference in the areas is $20.25\pi$, or about 64. If you answered **F**, you may have substituted the difference between the radii into the area formula, resulting in $2.25\pi$, or about 7. If you answered **G**, you may have substituted the difference in the diameters into the area formula, resulting in $9\pi$, or about 28. If you answered **H**, you may have found 25% of the area of the second circle, $0.25 \times \pi(7.5)^2$, or about 44. If you answered **K**, you may have substituted the diameters in for the radii in the area formula, resulting in $144\pi$ and $225\pi$. The difference is $81\pi$, or about 254.

**Question 45. The correct answer is D.** The first six prime numbers are 2, 3, 5, 7, 11, and 13. The median is the average of the middle two numbers 5 and 7, which is 6. The mean is the sum of the numbers, 41, divided by the number of terms, 6, which is 6.83. The product of 6 and 6.83 is 41. If you answered **A**, you may have used the numbers 2, 3, 4, 5, 6, and 7. The median is 4.5, and you multiplied by the number of items, resulting in 27. If you answered **B**, you may have used the numbers 2, 3, 5, 7, 9, and 11. The median is 6 and the mean is $\frac{37}{6}$. The product of the mean and the median is 37. If you answered **C**, you may have used the numbers 2, 4, 6, 7, 9, and 11. The median is the average of 6 and 7, or 6.5. Multiplying by the number of items results in 39. If you answered **E**, you may have used the first six even numbers 2, 4, 6, 8, 10, and 12. The median is 7 and the mean is 6. The product of 7 and 6 is 42.

**Question 46. The correct answer is K.** By the Pythagorean identity, $\sin^2(7x)+\cos^2(7x)=1$ is true for all real values of $x$. If you chose **F**, you determined that for all real values of $x$, $\sin(7x) + \cos(7x) = 7$ even though $\sin(7(0)) + \cos(7(0)) = 0 + 1 = 1$. If you chose **G**, you determined that for all real values of $x$, $\sin(7x) + \cos(7x) = 7$ even though $\sin(7\pi)+\cos(7\pi)=0+(-1)=-1$. If you chose **H**, you determined that for all real values of $x$, $7\sin(7x) + \cos(7x) = 14$ even though $7\sin(7(0)) + 7\cos(7(0)) = 7(0) + 7(1) = 7$. If you chose **J**, you determined that for all real values of $x$, $\sin^2(7x)+\cos^2(7x)=7$ even though $\sin(7(0)) * \sin(7(0)) + \cos(7(0)) * \cos(7(0)) = 0(0) + 1(1) = 1$.

**Question 47. The correct answer is A.** Because $\overline{AB}$ is a diameter, $\angle ADB$ is a right angle. Therefore, $\overline{AB}$ is a hypotenuse of $\Delta ADB$ and, by the Pythagorean theorem, has a length of $\sqrt{5^2+12^2}=13$ meters. Because the measure of angle $DAC$ is congruent to the measure of angle $DAB$, and angle $ADB$ and angle $DCA$ are both right angles, then triangle $ACD$ and triangle $DBC$ are similar triangles. Therefore, $\frac{CD}{AD}=\frac{BD}{AB}$, which is equivalent to $\frac{CD}{5}=\frac{12}{13}$. Therefore, $CD=\frac{60}{13}$. If you chose **B**, you may have computed $\frac{(AB)(AD)}{BD}$. If you chose **C**, you may have computed $AB$. If you chose **D**, you may have computed $\frac{(AB)(BD)}{AD}$. If you chose **E**, you may have computed $(BD)(AD)$.

**Question 48. The correct answer is K.** Because $a > 0$, $|a| = a$. Because $b < 0$, $|b| = -b$. Therefore, $|a| - |b| = a + b$. If you chose **F**, you determined for all positive $a$ and negative $b$ that $|a| - |b| = |a - b|$ even though $|1| - |-1| = 0 \neq 2 = |1 - (-1)|$. If you chose **G**, you determined for all positive $a$ and negative $b$ that $|a| - |b| = |a + b|$ even though $|1| - |-2| = 3 \neq 1 = |1 + (-2)|$. If you chose **H**, you determined for all positive $a$ and negative $b$ that $|a| - |b| = |a| + |b|$ even though $|1| - |1| = 0 \neq 2 = |1| + |1|$. If you chose **J**, you determined for all positive $a$ and negative $b$ that $|1| - |-1| = 0 \neq 2 = 1 - (-1)$.

**Question 49. The correct answer is D.** Because $x < y < 4$ is the only restriction on $x$ and $y$, it is possible for $x = 3.4$ and $y = 3.6$. Then $x + y = 3.4 + 3.6 = 7$. We also know that $x + y < y + y < 4 + 4 = 8$. So, the greatest possible integer value of $x + y$ is 7. If you chose **A**, you may have determined that $x + y$ could not be positive. If you chose **B**, you may have thought that 3 was the biggest integer that $x + y$ could be. If you chose **C**, you may have thought that 4 was the biggest integer that $y$ could be. If you chose **E**, you may have thought **x** and **y** could each be 4.

**Question 50. The correct answer is G.** Because $y$ varies directly as the square of $x$, then $y = kx^2$ for some constant $k$. Because $y = 20$ when $x = 2$, $20 = k(2^2)$, which means that $k = 5$. So, when $x = 3$, $y = 5(3^2) = 45$. If you chose **F**, you may have calculated $3(20 + 2 + 3)$. If you chose **H**, you may have used the equation $y = kx$ instead of the equation $y = kx^2$. If you chose **J**, you may have used the equation $y = k + x$ instead of the equation $y = kx^2$. If you chose **K**, you may have used the equation $y = k - x^2$ instead of the equation $y = kx^2$.

**Question 51. The correct answer is D.** The general equation for an ellipse centered at (0, 0) is $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ where $a$ is the length of the semimajor (horizontal) axis and $b$ is the length of the semiminor (vertical) axis. The values for $a$ and $b$ can be found by checking the intersection points of the ellipse with each circle. Because the general equation of a circle with radius $r$ centered at (0, 0) is $x^2+y^2=r^2$, the larger circle has a radius of $\sqrt{25}=5$ and the smaller circle has a radius of $\sqrt{4}=2$. This means that the ellipse must intersect with these circles at key points of (5, 0) and (0, 2), corresponding with semimajor and semiminor axes lengths of $a=5$ and $b=2$, respectively. Using these as the parameters in the general ellipse equation results in $\frac{x^2}{25}+\frac{y^2}{4}=1$. If you chose **A** or **B**, you may have used the wrong parameters for the semiminor and semimajor axes. If you chose **C**, you may have not squared the denominators. If you chose **E**, you may have thought a factor of 4 was necessary to include in the denominator.

**Question 52. The correct answer is H.** Let the two unknown integers be $x$ and $y$. Because the median of the set is 82, conclude that $x\geq 82$ and $y\geq 82$ because $0<82$ and $12<82$. Then the set of integers is $\{0,12,82,x,y\}$. Write an equation to represent the mean: $\frac{0+12+82+x+y}{5}=52$. Solve for $x+y$: $\frac{0+12+82+x+y}{5}=52\rightarrow 94+x+y=260\rightarrow x+y=166$. Combine with one of the inequalities to eliminate one variable: $x+82\leq x+y=166\rightarrow x+82\leq 166\rightarrow x\leq 84$. The same can be done for $y$. Now there is both a lower and upper limit on the values of $x$ and $y$, meaning that both values are between 82 and 84, inclusive. The only choice that fits these limits is **H**. If you chose **F** or **G**, you may not have considered that these values would change the median to something other than 82. If you chose **J** or **K**, you may not have considered that these values would change either the mean or the median.

**Question 53. The correct answer is B.** By the given definition, the integer 6 has factors of 1, 2, and 3 with a sum of $1+2+3=6$. The integer 8 has factors of 1, 2, and 4 with a sum of $1+2+4=7$. The integer 10 has factors of 1, 2, and 5 with a sum of $1+2+5=8$. The integer 12 has factors of 1, 2, 3, 4, and 6 with a sum of $1+2+3+4+6=16$. The only one of these integers that fits the given criteria of a factor sum greater than the integer itself is 12 because $16>12$.

**Question 54. The correct answer is G.** Vanna walked 2 miles per hour for $\frac{10}{60}$ of an hour and covered a distance of $2\frac{\text{mile}}{\text{hour}}\times\frac{10}{60}\text{ hour}=\frac{1}{3}\text{ mile}$. Similarly, after her speed increased, she walked $3\frac{\text{mile}}{\text{hour}}\times\frac{5}{60}\text{ hour}=\frac{1}{4}\text{ mile}$. This is a total of $\frac{1}{3}\text{ mile}+\frac{1}{4}\text{ mile}=\frac{4}{12}\text{ mile}+\frac{3}{12}\text{ mile}=\frac{7}{12}\text{ mile}$. Because she covered this distance in 15 minutes, her average rate over the whole distance is $\frac{\frac{7}{12}\text{ mile}}{\frac{15}{60}\text{ hour}}=\frac{7}{12}\times\frac{60\text{ mile}}{15\text{ hour}}=\frac{7\text{ mile}}{3\text{ hour}}$.

If you chose **F**, you may have only found the average rate for the first part of Vanna's walk. If you chose **H**, you may have added denominators when adding fractions. If you chose **J**, you may have divided by an extra factor of 60. If you chose **K**, you may have divided the sum of the rates by 2 instead of weighting the average rate by the time spent on each part of the walk.

**Question 55. The correct answer is D.** Let Alani's height be $x$. Then Baahir's height is $\frac{7}{5}x$ and Connor's height is $\frac{3}{4}\left(\frac{7}{5}x\right)=\frac{21}{20}x$. Dividing Alani's height by Connor's height to find the ratio results in $\frac{x}{\frac{21}{20}x}=\frac{20}{21}$. If you chose **A** or **B**, you may have subtracted or added the corresponding parts of each ratio. If you chose **C** or **E**, you may have multiplied the two given ratios.

**Question 56. The correct answer is K.** A square root is equivalent to a $\frac{1}{2}$ power. A cube root is equivalent to a $\frac{1}{3}$ power. Nesting a $\frac{1}{2}$ power and a $\frac{1}{3}$ power gives a $\frac{1}{6}$ power ($\frac{1}{2}\cdot\frac{1}{3}=\frac{1}{6}$). The base of the power is $x^2$, so we have $(x^2)^{\frac{1}{6}}=x^{2\left(\frac{1}{6}\right)}=x^{\frac{1}{3}}$. **F**, **G**, **H**, and **J** are all equivalent to $x^{\frac{1}{3}}$. Clearly, **K** is not equivalent to $x^{\frac{1}{3}}$; it is equivalent to $(x^{\frac{1}{3}})^2$.

**Question 57. The correct answer is B.** Let $l$ be the length and $w$ be the width of the original rectangle. The area of the original rectangle is $lw$. Increasing something by 25% is equivalent to multiplying by 1 + 0.25. Decreasing something by 10% is equivalent to multiplying by 1 – 0.10. Therefore, $1.25l$ and $0.9w$ are the length and width, respectively, of the resulting rectangle. The area of the resulting rectangle is $(1.25l)(0.9w) = 1.125lw$. Multiplying by 1.125 is equivalent to increasing by 12.5%. If you answered **A**, you may have multiplied 25% and 10%. If you answered **C**, you may have subtracted 10% from 25%. If you answered **D**, you may have multiplied 25% and 100% – 10%. If you answered **E**, you may have added 25% and 10%.

**Question 58. The correct answer is F.** A sum of 7 can be obtained with 2 and 5, 5 and 2, 3 and 4, or 4 and 3. The probability of a 2 and a 5 is $\frac{1}{5}\times\left(\frac{1}{4}\right)=\frac{1}{20}$ because 1 of the 5 balls is a 2, and 1 of the 4 remaining balls is a 5. Similarly, the probability of a 5 and a 2 is $\frac{1}{20}$, the probability of a 3 and a 4 is $\frac{1}{20}$, and the probability of a 5 and a 2 is $\frac{1}{20}$. Adding the probabilities of each of these 4 ways to obtain a sum of 7 gives $4\left(\frac{1}{20}\right)=\frac{1}{5}$. If you answered **G**, you may have used 2, the number of balls selected, divided by 5, the total number of balls. If you answered **H**, you may have used 4, the number of balls that could be drawn first, to obtain a sum of 7 (2 or 3 or 4 or 5) divided by 5, the total number of balls. If you answered **J**, you may have used $\frac{4}{5}$ for the probability that the first selection could make a sum of 7 (as in **H**), used $\frac{1}{4}$ for the probability that the second selection would be the 1 ball out of the remaining 4 balls to complete the sum of 7, and then incorrectly added $\frac{4}{5}$ and $\frac{1}{4}$ instead of multiplying them $\left(\frac{4}{5}+\frac{1}{4}=\frac{4+1}{5+4}=\frac{5}{9}\right)$. If you answered **K**, you may have thought the probability for each of the 4 ways to obtain a sum of 7 is $\frac{1}{5}\left(\frac{1}{5}\right)=\frac{1}{25}$.

**Question 59. The correct answer is D.** A function whose graph has no $x$-intercepts has either a minimum value greater than 0 or a maximum value less than 0. The function $y = \sin x$ has a minimum value of $-1$. In order to shift the graph vertically enough so that the minimum value is greater than 0, you'd have to add a value greater than 1: $c > 1$. The function $y = \sin x$ has a maximum value of 1. In order to shift the graph vertically enough so that the maximum value is less than 0, you'd have to add a value less than $-1$: $c < -1$. If you answered **A**, you may have simply looked for the amplitude of $y = \sin x$. If you answered **B**, you may have found the correct bound values for $c$ but got the shading wrong. If you answered **C**, you may have considered only shifting the graph up so the minimum value was greater than 0 but not considered shifting the graph down so the maximum value is less than 0. If you answered **E**, you may have thought that the minimum or maximum value could be 0.

**Question 60. The correct answer is G.** There are 3 feet in 1 yard, so there are $3^2 = 9$ square feet in 1 square yard. Divide the number of square feet needed by 9 to get the number of square yards needed: $360 \div 9 = 40$. If you answered **F**, you may have divided by $12(2) = 24$ instead of $3^2 = 9$. If you answered **H**, you may have divided by $3(2) = 6$ instead of $3^2 = 9$. If you answered **J**, you may have divided by $2^2 = 4$ instead of $3^2 = 9$. If you answered **K**, you may have divided by 3 instead of 9.