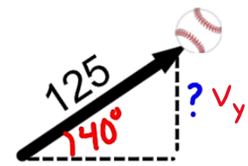
Name: ____ Per: ________

11.3 Derivative Application Worksheet

The equation for an object's height above the ground on Earth when moving with an initial upward velocity, v, in feet/second, is $h(t) = -16t^2 + vt + h_0$ where t is the time, in seconds, after it started, and h_0 is the initial height or height at time = 0, measured in feet.

a. A baseball is hit and leaves the bat going 125 feet/second at a 40° angle. Find the speed the ball is moving upward (round to the nearest foot/second).



$$\sin(40) = \frac{V_y}{125}$$

$$V_y = 125 \cdot \sin(40) = 80.34 \Rightarrow \boxed{80 \text{ ft/sec}}$$

b. If the ball was hit 4 feet above the ground, write the equation for the ball's height $h(t)$ by using this initial height of 4 ft and the initial upward velocity you found in part (a).

$$h(t) = -16t^2 + 80t + 4$$

c. Use your equation from part (b) to find the ball's height at the various times.

i. $t = 0\,sec$ $\quad h(0) = 4$ ft
ii. $t = 0.5\,sec$ $\quad h(0.5) = 40$ ft
iii. $t = 2\,sec$ $\quad h(2) = 100$ ft
iv. $t = 3\,sec$ $\quad h(3) = 100$ ft
v. $t = 4.5\,sec$ $\quad h(4.5) = 40$ ft

d. Recall from quadratics that the x-coordinate of the vertex is always located at $V_x = -b/2a$. Use this to find the maximum height of the ball and the time at which it reached this height.

$$V_x = \frac{-80}{2(-16)} = \frac{80}{32} = \boxed{2.5 \text{ sec}}$$

$$h(2.5) = \boxed{104 \text{ ft}}$$

e. How long was the ball in the air? (Hint: what is the height when it hits the ground and then how do you solve a quadratic equation = 0?) Height @ ground = 0

$$0 = -16t^2 + 80t + 4$$

use quadratic formula

$$t = \frac{-80 \pm \sqrt{80^2 - 4(-16)(4)}}{2(-16)}$$

$t = \cancel{-0.05}$

$\boxed{t = 5.05 \text{ sec}}$

f. Find a formula for the instantaneous rate of change for the ball's height (the derivative equation). If $h(t) = -16t^2 + 80t + 4$ then

$$h'(t) = \lim_{K \to 0} \frac{f(t+K) - f(t)}{K}$$

$$= \lim_{K \to 0} \frac{[-16(t+K)^2 + 80(t+K) + 4] - [-16t^2 + 80t + 4]}{K}$$

$$= \lim_{K \to 0} \frac{-16t^2 - 32tK - 16K^2 + 80t + 80K + 4 + 16t^2 - 80t - 4}{K}$$

$$= \lim_{K \to 0} \frac{-32tK - 16K^2 + 80K}{K}$$

$$= \lim_{K \to 0} -32t - 16K + 80$$

$$= -32t + 80$$

so $\boxed{h'(t) = -32t + 80}$

This is the velocity equation

$v(t) = -32t + 80$

CONTINUE ON BACK

g. Use your answer from part (f) to determine how fast the ball was moving upward at the following times.

i. $t = 0\ sec$ $h'(0) = v(0) = 80\ ft/sec$

ii. $t = 1\ sec$ $h'(1) = v(1) = 48\ ft/sec$

iii. $t = 3\ sec$ $h'(3) = v(3) = -16\ ft/sec$

iv. $t = 5\ sec$ $h'(5) = v(5) = -80\ ft/sec$

h. What is the ball's upward velocity when it reaches its maximum height? $0\ ft/sec$

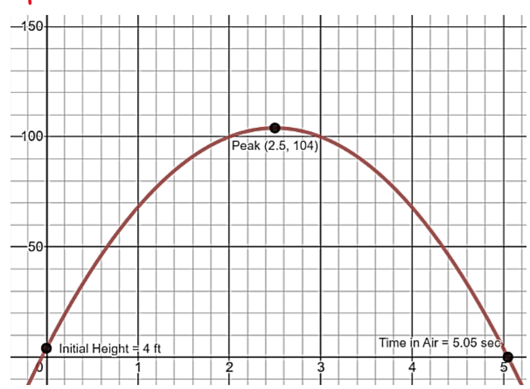
i. Use that hint to find the ball's maximum height and verify the answer you found in part (d).

When ball reaches peak, velocity = 0, so $h'(t)$ or $v(t) = 0$.

$0 = -32t + 80 \Rightarrow -80/-32 = t \Rightarrow t = 2.5$ sec at peak

$h(2.5) = 104$ ft! Same answer as in part d, we just used the derivative to prove it!

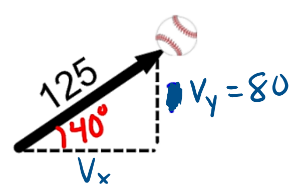
j. Use Desmos or a graphing calculator to sketch the equation for the ball's height you found in part (b) and verify all the answers you've found.



k. Find the initial horizontal velocity of the ball when it was hit.

$\cos(40) = \frac{V_x}{125} \Rightarrow V_x = 125\cos(40)$

$V_x = 95.8 \Rightarrow \boxed{96\ ft/sec}$



l. Use the time the ball was in the air that you found in part (e) along with the ball's horizontal velocity from part (k) to determine how far the ball traveled horizontally before hitting the ground. $d_x = v_x t = 96(5.05) = \boxed{484.8\ ft}$

m. If the ball was hit to center field at Yankee Stadium, does your answer to part (l) indicate this would be a home run? Support your claim.

An internet search shows that the distance to center field at Yankee Stadium is 408 ft, so it definitely has the horizontal distance to be a home run.

If you solve $d_x = v_x t$ for t when the distance is 408 feet, you get $t = \frac{408}{96} = 4.25\ sec$, which means the ball would have been going over the fence at 4.25 sec.

Plugging 4.25 sec into $h(t) = -16t^2 + 80t + 4$ gives $h(4.25) = 55\ ft$, meaning when the ball got to the fence it was 55 feet in the air, so it definitely cleared over the fence. HOME RUN!